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THEORETICAL ANALYSIS OF THE CREEP COLLAPSE OF COLUMNS

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SUMMARY

A theoretical analysis is made of the creep collapse of idealized H-section columns and solid rectangular-section columns by the application of a creep variational theorem. For the rectangular-section columns two assumptions are considered. In one case the stress distribution through the thickness is assumed to be linear. In the other case this stress distribution is allowed to take a nonlinear form. Charts are given for the critical lifetime parameters based on these various theories.

Comparisons are made between the theories for the rectangular-section columns and previously published test data. The comparisons indicate that the linear-stress theory gives nearly the same result as the nonlinear-stress theory. In addition, the refinement in stress distribution does not always improve the comparisons between theory and test. In many cases, the results obtained from the isochronous-tangent-modulus method are about as good as the results obtained from the more involved theoretical methods.

INTRODUCTION

The phenomenon of aerodynamic heating associated with the use of high-speed airplanes and missiles has motivated a number of studies of materials and structures at elevated temperatures. Among these studies are attempts to relate material creep properties to the creep behavior of structural compression elements. The present paper is concerned with this problem in regard to column behavior.

Design procedures are available for creep buckling. Methods which are entirely empirical predominate and ordinarily use static-buckling criteria with the tangent modulus replaced by its equivalent from isochronous stress-strain curves. Other approaches usually lead to methods which trace the displacement history starting with an initial imperfection.

Many papers have been written on column creep buckling. Results of a number of elevated-temperature creep tests on rectangular-section columns made of aluminum alloys are given in references 1 to 4. According to data in these papers, column lifetimes of apparently identical columns under identical loads may differ by a factor of two or three. Such wide scatter increases the difficulty of making clear-cut comparisons between theory and experiment.

The theoretical column-creep analyses presented in references 5 to 8 are displacement-type analyses. In references 5 and 6 the creep behavior of an idealized H-section column is analyzed by using a power creep law and tabulated results for column lifetime parameters are given. In references 7 and 8, H-section and solid rectangular-section columns are analyzed by using an exponential creep law which was modified in an attempt to account for creep behavior under varying stress. In all these displacement-type analyses, the differential equations were derived directly from statics. The length coordinate of the column was eliminated by assuming a deflected shape and by satisfying the differential equations at the column midheight only.

Approximate solutions which satisfy basic equations on an average over a region are obtained when variational methods are used. One variational theorem applicable to creep of columns is presented in reference 9. It yields as Euler equations the time derivatives of static equations of equilibrium, stress-strain relations, and boundary conditions. This variational theorem is analogous to the theorem for elasticity given in references 10 and 11. In reference 12 a derivation of this creep variational theorem is presented which starts with the theorem of references 10 and 11; in addition, the H-section column is analyzed in reference 12 by using the theorem, and little difference is found between the results obtained from the variational theorem and previous work in references 5 and 6 done by collocation.

The present paper is a further application of the creep variational theorem of reference 9 to the analysis of creep behavior of columns. The theorem is applied to a solid rectangular-section column in which a nonlinear stress distribution is permitted through the thickness. For comparison, solutions are also presented for the idealized H-section column and the rectangular-section column in which the stress distribution is constrained to remain linear through the thickness. Also included for purposes of comparison is an engineering method - the so-called isochronous-tangent-modulus method. Four sets of existing experimental data on column-creep tests and material creep properties from references 1 to 4 are included in a comparison with theory. The investigation was carried out to determine whether these various refinements improve the agreement between theory and experiment.

SYMBOLS

A	area of cross section
a_j	numerical-integration coefficient
B	constant in creep law
$B^* = \frac{BP}{A}$	
b	width of rectangular-section column
C	constant in creep law
c_j	nondimensional coordinate for numerical integration
E	Young's modulus
$f(\sigma)$	function of stress (see eq. (2))
$g(t)$	function of time (see eq. (2))
h	thickness of rectangular-section column
h_1	distance between flanges of idealized H-section column
I_1	modified Bessel function of first kind
k	constant in creep law
l	length of column
N	number of stations through thickness of column
n	constant in creep law
P	compressive load
P_E	Euler buckling stress of column
P_T	modified Euler buckling stress of column
s	lateral-displacement parameter for rectangular-section column, $6B^*W$

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s_1	lateral-displacement parameter for idealized H-section column, $2B^*W_1$
T	temperature
t	time
U	displacement coefficient in equation (4)
u	displacement of neutral surface in x-direction
V	volume
W	displacement coefficient in equation (3)
W_1	displacement coefficient in equation (7)
w	displacement of column in z-direction, with initial crookedness included
x,y,z	coordinates
α,β,γ	dummy integration variables
δ_{ij}	Kronecker delta
ϵ	strain, positive in tension
ϵ''	creep strain
Π	quantity to be varied
ρ	radius of gyration
σ	stress, positive in tension
$\bar{\sigma}$	average column stress, positive in compression
σ_B	difference between stress at a point in column and average stress on column, positive in compression
$(\sigma_{B,elem})_{max}$	maximum value of σ_B for cross section calculated by elementary beam theory
$\sigma_0,\sigma_1,\sigma_2,\sigma_3,\sigma_4,\sigma_{5j}$	stress coefficients

σ^* constant in creep law

τ_p time parameter when power law is used, $\left(\frac{l}{\pi\rho}\right)^2 \frac{n\left(\frac{P}{A\sigma^*}\right)^n}{1 - \frac{P}{P_E}} g$

τ_s time parameter when hyperbolic sine law is used,
$\left(\frac{l}{\pi\rho}\right)^2 \frac{CB^* \cosh B^*}{1 - \frac{P}{P_E}} g$

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$\varphi_0, \varphi_1, \varphi_2, \varphi_3, \varphi_{4j}, \varphi_{5j}$ integrals involving the creep law

$\varphi_{5j}^* = \left(\frac{A\sigma^*}{P}\right)^n \frac{\varphi_{5j}}{n}$

$\psi\left(\frac{z}{h_1}\right)$ function of z/h_1, defined by equation (6)

Subscripts and superscripts:

cr critical

i,j integers

o built-in or initial crookedness

p power creep law

s hyperbolic sine creep law

x differentiation with respect to x

Dots over quantities denote time derivatives.

ANALYSIS

General Discussion

In order to exhibit the various stages of refinement possible in an analysis of the creep collapse of columns by using variational methods, several problems are considered. The analysis is applied to idealized H-section columns and solid rectangular-section columns by means of a creep variational theorem. The idealized H-section column is treated first, and then consideration is given to a solid rectangular-section column where the stress distribution is required to vary in a linear fashion through the column thickness. For both the H-section column and the solid rectangular-section column, solutions are presented for the two creep laws - the hyperbolic sine law and the power law. Finally, the solid rectangular-section column is treated by allowing the stress distribution to be nonlinear through the thickness. In this case the solution using the hyperbolic sine creep law becomes rather involved, and only the solution with the power law is carried to completion. Throughout these analyses the collocation approach has been abandoned and has been replaced by the more refined variational theorem.

A Rayleigh-Ritz procedure is used in conjunction with the creep variational theorem of reference 9. It is assumed that the strain distribution through the column thickness is linear. The load is assumed to be applied quickly enough so that no creep occurs while loading, yet slowly enough so that dynamic effects may be neglected. The load is then assumed to remain constant until the column collapses. The variational theorem states that the expression

$$\Pi = \int_V \left[\dot{\sigma}\left(\dot{u}_x + w_x\dot{w}_x - z\dot{w}_{xx}\right) + \frac{\sigma}{2}\,\dot{w}_x^{\,2} - \frac{\dot{\sigma}^2}{2E} - \dot{\sigma}\dot{\epsilon}''\right]dV \tag{1}$$

when varied with respect to the time derivatives of quantities yields the desired column creep equations.

The material creep relations used in this report have the form

$$\dot{\epsilon}'' = f(\sigma)\ \dot{g}(t) \tag{2}$$

where $f(\sigma)$ is a function of stress alone and $\dot{g}(t)$ is a function of time alone. These relations apply only for constant temperature and constant stress conditions. In this investigation, however, it is assumed that these creep relations apply also in the case of varying stress. For the hyperbolic sine law the function $f(\sigma)$ is given by

$$f(\sigma) = C \sinh B\sigma$$

where B and C are material constants. The power law gives $f(\sigma)$ as

$$f(\sigma) = \left(\frac{\sigma}{\sigma^*}\right)^n$$

where σ^* and n are material constants. The function $g(t)$ provides an approximate way to account for primary creep. In the calculations made in this investigation, g was taken in the form

$$g(t) = t^k$$

$$\dot{g}(t) = kt^{k-1}$$

where k is a material constant.

The lateral displacements of the middle surface of the column are illustrated in figure 1(a). The symbol w_o represents the initial lateral displacement or initial crookedness of the column. The lateral displacement at the instant after load is applied and before creep begins is represented by $w(0)$. The symbol w represents the lateral displacement as creep progresses. The initial crookedness is included in $w(0)$ and w. It is assumed that a sufficiently accurate approximation to the displacement shape of the column is given by a sine curve

$$w = h\,W(t)\,\sin\frac{\pi x}{l} \tag{3}$$

The displacements of points on the neutral surface in the x-direction are assumed to be given by

$$u = x\,U(t) \tag{4}$$

Idealized H-Section Column

Consider the idealized H-section column shown in figure 1(b). This column has two flanges each of area $\frac{A}{2}$ separated a distance h_1 by a web flexible in bending in its own plane. The stresses are assumed to be given by

$$\sigma = \frac{P}{A}\left[\sigma_0 + \sigma_1\,\psi\left(\frac{z}{h_1}\right)\sin\frac{\pi x}{l}\right] \tag{5}$$

where

$$\left.\begin{aligned}\psi\left(\pm\frac{1}{2}\right) &= \pm\frac{1}{2} \\ \psi\left(\frac{z}{h_1}\right) &= 0 \qquad \left(\frac{z}{h_1} \neq \pm\frac{1}{2}\right)\end{aligned}\right\} \tag{6}$$

The displacements are given by equations (3) and (4) with h and W replaced by h_1 and W_1. The quantity to be varied becomes

$$\Pi = P\int_0^l \left\{\dot{\sigma}_0\left[\dot{U} + W_1\dot{W}_1\left(\frac{\pi h_1}{l}\right)^2 \cos^2\frac{\pi x}{l}\right] + \dot{\sigma}_1\dot{W}_1\left(\frac{\pi h_1}{l}\right)^2 \frac{1}{4}\sin^2\frac{\pi x}{l}\right.$$

$$\left. + \frac{\sigma_0}{2}\dot{W}_1^2\left(\frac{\pi h_1}{l}\right)^2\cos^2\frac{\pi x}{l} - \frac{P}{2EA}\left(\dot{\sigma}_0^2 + \frac{\dot{\sigma}_1^2}{4}\sin^2\frac{\pi x}{l}\right)\right\} dx$$

$$- \frac{P}{A}\dot{\sigma}_0\dot{g}\int_V f(\sigma)\,dV - \frac{P}{A}\dot{\sigma}_1\dot{g}\int_V \psi\left(\frac{z}{h_1}\right)\sin\frac{\pi x}{l}\,f(\sigma)\,dV \qquad (7)$$

When the integrations are performed and the nondimensional quantities

$$\left.\begin{aligned} \varphi_0 &= \frac{1}{Al}\int_V f(\sigma)\,dV \\ \varphi_1 &= \frac{1}{Al}\int_V \psi\left(\frac{z}{h_1}\right)\sin\frac{\pi x}{l}\,f(\sigma)\,dV \end{aligned}\right\} \qquad (8)$$

are introduced, equation (7) becomes

$$\Pi = Pl\left\{\dot{\sigma}_0\left[\dot{U} + W_1\dot{W}_1\left(\frac{\pi h_1}{l}\right)^2\frac{1}{2}\right] + \dot{\sigma}_1\dot{W}_1\left(\frac{\pi h_1}{l}\right)^2\frac{1}{8}\right.$$

$$\left. + \sigma_0\dot{W}_1^2\left(\frac{\pi h_1}{l}\right)^2\frac{1}{4} - \frac{P}{2EA}\dot{\sigma}_0^2 - \frac{P\dot{\sigma}_1^2}{16EA} - \dot{\sigma}_0\dot{g}\varphi_0 - \dot{\sigma}_1\dot{g}\varphi_1\right\} \qquad (9)$$

The appropriate equations for the column are obtained by setting equal to zero the partial derivatives $\frac{\partial\Pi}{\partial\dot{U}}$, $\frac{\partial\Pi}{\partial\dot{W}_1}$, $\frac{\partial\Pi}{\partial\dot{\sigma}_0}$, and $\frac{\partial\Pi}{\partial\dot{\sigma}_1}$. Thus, the equations obtained are

$$\dot{\sigma}_0 = 0 \qquad (10a)$$

$$\dot{\sigma}_0 W_1\frac{1}{2} + \dot{\sigma}_1\frac{1}{8} + \sigma_0\dot{W}_1\frac{1}{2} = 0 \qquad (10b)$$

$$\frac{EA}{P}\dot{U} + 2W_1\dot{W}_1\frac{P_E}{P} - \dot{\sigma}_0 - \dot{g}\frac{EA}{P}\varphi_0 = 0 \qquad (10c)$$

$$\dot{W}_1 \frac{P_E}{P} - \frac{\dot{\sigma}_1}{4} - 2 \frac{EA}{P} \dot{g}\varphi_1 = 0 \tag{10d}$$

If U is not required, equation (10c) may be ignored; then, equations (10a), (10b), and (10d) may be combined with the use of the elastic initial conditions

$$W_1(0) = \frac{W_{1,o}}{1 - \frac{P}{P_E}} \tag{11a}$$

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$$\sigma_0(0) = -1 \tag{11b}$$

$$\sigma_1(0) = 4W_1(0) \tag{11c}$$

to form the differential equation

$$\left(\frac{\pi\rho}{l}\right)^2\left(1 - \frac{P}{P_E}\right)\frac{dW_1}{dg} = 2\varphi_1 \tag{12}$$

The creep law must be prescribed in order to obtain solutions to equation (12). For the hyperbolic sine law which is given as

$$\dot{\epsilon}_s = \frac{\dot{\sigma}}{E} + \dot{g}C \sinh B\sigma \tag{13}$$

the function of stress is

$$f(\sigma) = C \sinh B\sigma \tag{14}$$

and the required integral involving this function is

$$\varphi_1 = \frac{1}{Al}\int_A\int_0^l C\,\psi\left(\frac{z}{h_1}\right)\sin\frac{\pi x}{l}\sinh\left\{B^*\left[-1 + \psi\left(\frac{z}{h_1}\right)4W_1 \sin\frac{\pi x}{l}\right]\right\}dx\,dA$$

$$= \frac{C}{2}\cosh B^*\, I_1\left(2B^*W_1\right) \tag{15}$$

where I_1 is the modified Bessel function of the first kind and $B^* = \frac{BP}{A}$. With use of the nondimensional parameters

$$\left.\begin{aligned} s_1 &= 2B^{*}W_1 \\ \tau_s &= \left(\frac{l}{\pi\rho}\right)^2 \frac{CB^{*}\cosh B^{*}}{1 - \frac{P}{P_E}} g \end{aligned}\right\} \tag{16}$$

equation (12) may be written as

$$\frac{ds_1}{d\tau_s} = 2I_1(s_1) \tag{17}$$

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The time parameter τ_s as a function of the lateral-displacement parameter s_1 is

$$\tau_s = \frac{1}{2} \int_{s_1(0)}^{s_1} \frac{d\alpha}{I_1(\alpha)} \tag{18}$$

where the parameter $s_1(0)$ is given by

$$s_1(0) = \frac{2B^{*}W_{1,o}}{1 - \frac{P}{P_E}} \tag{19}$$

The critical time parameter is

$$\tau_{s,cr} = \frac{1}{2} \int_{s_1(0)}^{\infty} \frac{d\alpha}{I_1(\alpha)} \tag{20}$$

The integration indicated in this equation can be carried out readily by numerical methods.

The analysis for the power creep law is given here for the sake of completeness, although results have been presented already in reference 12. The function of stress is

$$f(\sigma) = \left(\frac{\sigma}{\sigma^{*}}\right)^n \tag{21}$$

In the cases considered herein, n will be restricted to odd integers in order to simplify integrations. The integral required with this creep law is

$$\varphi_1 = \left(\frac{P}{A\sigma^*}\right)^n \frac{1}{Al} \int_A \int_0^l \psi\left(\frac{z}{h_1}\right) \sin \frac{\pi x}{l} \left[-1 + \psi\left(\frac{z}{h_1}\right) 4W_1 \sin \frac{\pi x}{l}\right]^n dx\, dA$$

$$= \left(\frac{P}{A\sigma^*}\right)^n \frac{1}{4} \sum_{i=1,3}^{n} \frac{n!(i + 1)W_1{}^i}{(n - i)!\left[\left(\frac{i + 1}{2}\right)!\right]^2} \tag{22}$$

The differential equation obtained for the idealized H-section column with the power creep law is

$$\frac{dW_1}{d\tau_p} = \frac{1}{2} \sum_{i=1,3}^{n} \frac{(n - 1)!(i + 1)W_1{}^i}{(n - i)!\left[\left(\frac{i + 1}{2}\right)!\right]^2} \tag{23}$$

where

$$\tau_p = \left(\frac{l}{\pi\rho}\right)^2 \frac{n\left(\frac{P}{A\sigma^*}\right)^n}{1 - \frac{P}{P_E}} g \tag{24}$$

and the critical time parameter is

$$\tau_{p,cr} = \int_{W_1(0)}^{\infty} \frac{dW_1}{\frac{1}{2} \sum_{i=1,3}^{n} \frac{(n - 1)!(i + 1)W_1{}^i}{(n - i)!\left[\left(\frac{i + 1}{2}\right)!\right]^2}} \tag{25}$$

where $W_1(0)$ is given by equation (11a). Evaluation of the integral in equation (25) can be carried out easily for $n = 3$ and $n = 5$. Numerical integration is more convenient for larger values of n.

An idealized H-section column having the same cross-sectional area and moment of inertia as a given rectangular-section column is called an equivalent H-section column in this paper. For this case, $h_1 = \frac{h}{\sqrt{3}}$ and, of course, each flange has one-half the area of the rectangular-section column. Note, also, that for the equivalent H-section column $W_1 = \sqrt{3}\, W$ and $s_1 = \frac{s}{\sqrt{3}}$.

Solid Rectangular-Section Column With Linear Stress Distribution Through the Thickness

Consider a solid rectangular-section column of width b, thickness h, and length l as shown in figure 1(c). Assume that stress as well as strain varies linearly through the thickness. The stresses are assumed to be given by

$$\sigma = \frac{P}{A}\left(\sigma_2 + \sigma_3 \frac{z}{h} \sin \frac{\pi x}{l}\right) \tag{26}$$

For the stresses given by equation (26) and the displacements given by equations (3) and (4), the quantity to be varied is

$$\begin{aligned}
\Pi = \frac{Pb}{A} \int_{-h/2}^{h/2} \int_0^l \Bigg\{ & \dot{\sigma}_2 \left[\dot{U} + W\dot{W}\left(\frac{\pi h}{l}\right)^2 \cos^2 \frac{\pi x}{l}\right] + \dot{\sigma}_3 \dot{W}\left(\frac{\pi h}{l}\right)^2 \left(\frac{z}{h}\right)^2 \sin^2 \frac{\pi x}{l} \\
& + \frac{\sigma_2}{2} \dot{W}^2 \left(\frac{\pi h}{l}\right)^2 \cos^2 \frac{\pi x}{l} - \frac{P}{2EA}\left[\dot{\sigma}_2{}^2 + \dot{\sigma}_3{}^2 \left(\frac{z}{h}\right)^2 \sin^2 \frac{\pi x}{l}\right]\Bigg\} dx\, dz \\
& - \frac{P}{A}\, \dot{\sigma}_2 \dot{g} \int_V f(\sigma)\, dV - \frac{P}{A}\, \dot{\sigma}_3 \dot{g} \int_V \frac{z}{h} \sin \frac{\pi x}{l}\, f(\sigma)\, dV
\end{aligned} \tag{27}$$

After integrations are performed and the functions

$$\left.\begin{aligned}
\varphi_2 &= \frac{1}{Al} \int_V f(\sigma)\, dV \\
\varphi_3 &= \frac{1}{Al} \int_V \frac{z}{h} \sin \frac{\pi x}{l}\, f(\sigma)\, dV
\end{aligned}\right\} \tag{28}$$

are introduced, equation (27) becomes

$$\begin{aligned}
\Pi = Pl \Bigg\{ & \dot{\sigma}_2 \left[\dot{U} + W\dot{W}\left(\frac{\pi h}{l}\right)^2 \frac{1}{2}\right] + \dot{\sigma}_3 \dot{W}\left(\frac{\pi h}{l}\right)^2 \frac{1}{24} + \sigma_2 \dot{W}^2 \left(\frac{\pi h}{l}\right)^2 \frac{1}{4} \\
& - \frac{P\dot{\sigma}_2{}^2}{2EA} - \frac{P\dot{\sigma}_3{}^2}{48EA} - \dot{\sigma}_2 \dot{g} \varphi_2 - \dot{\sigma}_3 \dot{g} \varphi_3 \Bigg\}
\end{aligned} \tag{29}$$

As before, the column equations are obtained by equating to zero the partial derivatives $\frac{\partial\Pi}{\partial\dot{U}}$, $\frac{\partial\Pi}{\partial\dot{W}}$, $\frac{\partial\Pi}{\partial\dot{\sigma}_2}$, and $\frac{\partial\Pi}{\partial\dot{\sigma}_3}$. Thus, the following equations are obtained:

$$\dot{\sigma}_2 = 0 \tag{30a}$$

$$\dot{\sigma}_2 W \frac{1}{2} + \dot{\sigma}_3 \frac{1}{24} + \sigma_2 \dot{W} \frac{1}{2} = 0 \tag{30b}$$

$$\frac{EA}{P} \dot{U} + 6\, W\dot{W} \frac{P_E}{P} - \dot{\sigma}_2 - \frac{EA}{P} \dot{g}\varphi_2 = 0 \tag{30c}$$

$$\dot{W} \frac{P_E}{P} - \dot{\sigma}_3 \frac{1}{12} - 2 \frac{EA}{P} \dot{g}\varphi_3 = 0 \tag{30d}$$

Initial conditions for this problem are

$$\left.\begin{aligned} W(0) &= \frac{W_o}{1 - \frac{P}{P_E}} \\ \sigma_2(0) &= -1 \\ \sigma_3(0) &= 12W(0) \end{aligned}\right\} \tag{31}$$

Equation (30c) may be ignored unless the calculation of U is desired. Equations (30a), (30b), and (30d) may be combined with equations (31) to give

$$\left(\frac{\pi\rho}{l}\right)^2 \left(1 - \frac{P}{P_E}\right)\frac{dW}{dg} = 2\varphi_3 \tag{32}$$

When the hyperbolic sine creep law (eq. (13)) is used, φ_3 is written as

$$\begin{aligned} \varphi_3 &= \frac{C}{hl} \int_0^l \int_{-h/2}^{h/2} \frac{z}{h} \sin\frac{\pi x}{l} \sinh\left[B^*\left(-1 + 12W \frac{z}{h} \sin\frac{\pi x}{l}\right)\right] dz\, dx \\ &= C \cosh B^* \frac{1}{2s^2} \int_0^s \beta\, I_1(\beta)\, d\beta \end{aligned} \tag{33}$$

where

$$\left.\begin{aligned} s &= 6B^* W \\ \beta &= s\frac{z}{h} \end{aligned}\right\} \tag{34}$$

and where $I_1(\beta)$ is the modified Bessel function of the first kind of order one. With the use of equation (33) and the definition for τ_s in equation (16), equation (32) becomes

$$\frac{ds}{d\tau_s} = \frac{6\int_0^s \beta\ I_1(\beta)\ d\beta}{s^2} \tag{35}$$

The critical time parameter for the rectangular-section column having linear stress distribution through the thickness and following the hyperbolic sine creep law is

$$\tau_{s,cr} = \frac{1}{6}\int_{s(0)}^{\infty} \frac{\gamma^2 d\gamma}{\int_0^{\gamma} \beta\ I_1(\beta)\ d\beta} \tag{36}$$

where

$$s(0) = \frac{6B^* W_o}{1 - \frac{P}{P_E}} \tag{37}$$

When the power creep law is used, φ_3 is written as

$$\begin{aligned}\varphi_3 &= \left(\frac{P}{A\sigma^*}\right)^n \frac{1}{hl} \int_{-h/2}^{h/2}\int_0^l \frac{z}{h} \sin\frac{\pi x}{l}\left(-1 + 12W\frac{z}{h}\sin\frac{\pi x}{l}\right)^n dx\ dz \\ &= \left(\frac{P}{A\sigma^*}\right)^n \frac{1}{4} \sum_{i=1,3}^{n} \frac{n!(i + 1)(3W)^i}{(n - i)!\left[\left(\frac{i + 1}{2}\right)!\right]^2 (i + 2)}\end{aligned} \tag{38}$$

and equation (32) becomes

$$\frac{dW}{d\tau_{p,cr}} = \frac{1}{2} \sum_{i=1,3}^{n} \frac{(n - 1)!(i + 1)(3W)^i}{(n - i)!\left[\left(\frac{i + 1}{2}\right)!\right]^2 (i + 2)} \tag{39}$$

The initial condition is

$$W(0) = \frac{W_o}{1 - \frac{P}{P_E}} \tag{40}$$

and the critical time parameter is

$$\tau_{p,cr} = \int_{W(0)}^{\infty} \frac{dW}{\frac{1}{2}\sum\limits_{i=1,3}^{n} \frac{(n - 1)!(i + 1)(3W)^{i}}{(n - i)!\left[\left(\frac{i + 1}{2}\right)!\right]^{2}(i + 2)}} \tag{41}$$

Equations (36) and (41) can be solved in the same way as were equations (20) and (25).

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Solid Rectangular-Section Column With Nonlinear Stress

Distribution Through the Thickness

In the previous solutions, the shape of the stress distribution is fixed from the beginning. For the case treated in this section a rectangular-section column is analyzed by allowing the stress distribution through the thickness to be arbitrary. In the quantity to be varied in the variational theorem (eq. (1)) the integrals through the column thickness are replaced by summations corresponding to numerical integrations. The equations which follow are set up on the basis of the Gaussian quadrature formula. (See ref. 13.) Similar results would be obtained by using other quadrature formulas.

The z-coordinate distance from the centroidal axis to any station j across the thickness is hc_j. The stress at a distance hc_j from the centroidal axis is

$$\sigma_j = \frac{P}{A}\left(\sigma_4 + \sigma_{5j} \sin \frac{\pi x}{l}\right) \tag{42}$$

and displacements are given by equations (3) and (4). The quantity to be varied becomes

$$\begin{aligned}\Pi = P \sum_{j=1}^{N} a_j \int_0^{l} \Bigg\{ & \left(\dot{\sigma}_4 + \dot{\sigma}_{5j} \sin \frac{\pi x}{l}\right)\left[\dot{U} + W\dot{W}\left(\frac{\pi h}{l}\right)^2 \cos^2\frac{\pi x}{l} + \dot{W}\left(\frac{\pi h}{l}\right)^2 c_j \sin \frac{\pi x}{l}\right] \\ & + \frac{1}{2}\left(\sigma_4 + \sigma_{5j} \sin \frac{\pi x}{l}\right)\dot{W}^2\left(\frac{\pi h}{l}\right)^2 \cos^2\frac{\pi x}{l} - \frac{P}{2EA}\left(\dot{\sigma}_4{}^2 + 2\dot{\sigma}_4\dot{\sigma}_{5j} \sin \frac{\pi x}{l} + \dot{\sigma}_{5j}{}^2 \sin^2\frac{\pi x}{l}\right) \\ & - \dot{g}\left(\dot{\sigma}_4 + \dot{\sigma}_{5j} \sin \frac{\pi x}{l}\right) f(\sigma_j)\Bigg\} dx\end{aligned} \tag{43}$$

where a_j represents the Gaussian quadrature coefficients. When some of the integrations over x are performed, the result is

$$\Pi = Pl \sum_{j=1}^{N} a_j \Bigg\{ \dot{\sigma}_4 \left[\dot{U} + W\dot{W}\left(\frac{\pi h}{l}\right)^2 \frac{1}{2} \right] + \dot{\sigma}_{5j} \left[\dot{U}\,\frac{2}{\pi} + W\dot{W}\left(\frac{\pi h}{l}\right)^2 \frac{2}{3\pi} + \dot{W}\left(\frac{\pi h}{l}\right)^2 c_j\,\frac{1}{2} \right]$$

$$+ \left(\sigma_4\,\frac{1}{4} + \sigma_{5j}\,\frac{1}{3\pi}\right)\dot{W}^2\left(\frac{\pi h}{l}\right)^2 - \frac{P}{EA}\left(\dot{\sigma}_4{}^2\,\frac{1}{2} + \dot{\sigma}_4\dot{\sigma}_{5j}\,\frac{2}{\pi} + \dot{\sigma}_{5j}{}^2\,\frac{1}{4}\right)$$

$$- \dot{\sigma}_4\dot{g}\varphi_{4j} - \dot{\sigma}_{5j}\dot{g}\varphi_{5j} \Bigg\} \tag{44}$$

where

$$\left.\begin{aligned} \varphi_{4j} &= \frac{1}{l}\int_0^l f(\sigma_j)\,dx \\ \varphi_{5j} &= \frac{1}{l}\int_0^l \sin\frac{\pi x}{l}\, f(\sigma_j)\,dx \end{aligned}\right\} \tag{45}$$

The remainder of the analysis is simplified if it is assumed at this point that the average stress on the cross section is independent of time; that is,

$$\dot{\sigma}_4 = 0$$

and

$$\sigma_4 = -1$$

Equating to zero the variation of Π with respect to $\dot{U}$, $\dot{W}$, and $\dot{\sigma}_{5j}$ is equivalent to equating to zero the partial derivatives of Π with respect to $\dot{U}$, $\dot{W}$, and $\dot{\sigma}_{5j}$. Use of this procedure yields the following equations:

$$\sum_{j=1}^{N} a_j\dot{\sigma}_{5j} = 0 \tag{46a}$$

$$\sum_{j=1}^{N} a_j\left[\dot{\sigma}_{5j} W \frac{2}{3\pi} + \dot{\sigma}_{5j} c_j \frac{1}{2} + \left(\sigma_4 \frac{1}{2} + \sigma_{5j} \frac{2}{3\pi}\right)\dot{W}\right] = 0 \tag{46b}$$

$$\dot{U} \frac{2}{\pi} + W\dot{W}\left(\frac{\pi h}{l}\right)^2 \frac{2}{3\pi} + \dot{W}\left(\frac{\pi h}{l}\right)^2 c_j \frac{1}{2} - \frac{P}{EA} \dot{\sigma}_{5j} \frac{1}{2} - \dot{g}\varphi_{5j} = 0 \tag{46c}$$

Equations (46) may be solved for $\dot{W}$ and $\dot{\sigma}_{5j}$ in terms of φ_{5j}. In order to accomplish this solution, the following properties of the Gaussian quadrature coefficients are used:

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$$\left.\begin{aligned} \sum_{j=1}^{N} a_j &= 1 \\ \sum_{j=1}^{N} a_j c_j &= 0 \\ \sum_{j=1}^{N} a_j {c_j}^2 &= \frac{1}{12} \end{aligned}\right\} \tag{47}$$

If equation (46c) (representing N equations) is multiplied by a_j and summed over j from 1 to N, the result is

$$\dot{U} \frac{2}{\pi} + W\dot{W}\left(\frac{\pi h}{l}\right)^2 \frac{2}{3\pi} = \dot{g} \sum_{j=1}^{N} a_j \varphi_{5j} \tag{48}$$

Equations (48) and (46c) may be solved for $\dot{\sigma}_{5j}$ with the following result:

$$\dot{\sigma}_{5j} = \frac{2EA}{P}\left[\dot{W}\left(\frac{\pi h}{l}\right)^2 c_j \frac{1}{2} + \dot{g} \sum_{i=1}^{N} \left(a_i - \delta_{ij}\right)\varphi_{5i}\right] \tag{49}$$

Equation (46b) may be written as

$$\dot{W} = \sum_{j=1,2}^{N} a_j c_j \dot{\sigma}_{5j} \tag{50}$$

With the use of equation (49), equation (50) becomes

$$\left(\frac{\pi\rho}{l}\right)^2\left(1 - \frac{P}{P_E}\right)\dot{W} = 2\dot{g} \sum_{j=1,2}^{N} a_j c_j \varphi_{5j} \tag{51}$$

Consequently, equation (49) may be given in the form

$$\left(\frac{\pi\rho}{l}\right)^2\left(1 - \frac{P}{P_E}\right)\dot{\sigma}_{5j} = 2\dot{g}\left(\frac{P_E}{P} - 1\right) \sum_{i=1,2}^{N} \left[\left(1 + \frac{12c_i c_j}{1 - \frac{P}{P_E}}\right)a_i - \delta_{ij}\right]\varphi_{5i} \tag{52}$$

If the power law is taken as the creep relation, the integral φ_{5j} in equation (52) is

$$\begin{aligned}
\varphi_{5j} &= \left(\frac{P}{A\sigma^*}\right)^n \frac{1}{l}\int_0^l \sin\frac{\pi x}{l} \sum_{i=0,1}^{n} \binom{n}{i}(-1)^{n-i}\left(\sigma_{5j}\sin\frac{\pi x}{l}\right)^i dx \\
&= \left(\frac{P}{A\sigma^*}\right)^n \left\{\frac{1}{2}\sum_{i=1,3}^{n} \frac{n!(i + 1)\left(\frac{\sigma_{5j}}{2}\right)^i}{(n - i)!\left[\left(\frac{i + 1}{2}\right)!\right]^2}\right. \\
&\qquad \left. - \frac{2}{\pi} - \frac{2}{\pi}\sum_{i=2,4}^{n-1} \frac{n!\left[\left(\frac{i}{2}\right)!\right]^2(2\sigma_{5j})^i}{(n - i)!\,i!\,(i + 1)!}\right\}
\end{aligned} \tag{53}$$

With

$$\left.\begin{aligned}
\tau_p &= \left(\frac{l}{\pi\rho}\right)^2 \frac{n\left(\frac{P}{A\sigma^*}\right)^n}{1 - \frac{P}{P_E}}\,g \\
\varphi_{5j}^* &= \left(\frac{A\sigma^*}{P}\right)^n \frac{\varphi_{5j}}{n}
\end{aligned}\right\} \tag{54}$$

equations (51) and (52) become, respectively,

$$\frac{dW}{d\tau_p} = 2 \sum_{j=1,2}^{N} a_j c_j \varphi_{5j}{}^* \tag{55}$$

$$\frac{d\sigma_{5j}}{d\tau_p} = 2\left(\frac{P_E}{P} - 1\right) \sum_{i=1,2}^{N} \left[\left(1 + \frac{12c_i c_j}{1 - \frac{P}{P_E}}\right)a_i - \delta_{ij}\right]\varphi_{5i}{}^* \tag{56}$$

Initial conditions for this case are

$$\left.\begin{aligned} W(0) &= \frac{W_o}{1 - \frac{P}{P_E}} \\ \sigma_{5j}(0) &= 12c_j W(0) \end{aligned}\right\} \tag{57}$$

Equations (55) to (57) give $N + 1$ first-order differential equations and their initial conditions. These differential equations may be solved by a numerical method. When $N = 2$ and Gaussian quadrature formulas are used, equations (55) and (56) reduce to equation (23) which in this case applies to an idealized H-section column having the same area and moment of inertia as the rectangular section, that is, the equivalent H-section column.

Isochronous-Tangent-Modulus Method

An engineering method which has been used for predicting column creep collapse is the so-called isochronous-tangent-modulus method. In this method it is assumed that collapse occurs when the stress corresponding to the tangent modulus from an isochronous stress-strain curve becomes equal to the average column stress. If the column material follows the creep law which is given as

$$\epsilon = \frac{\sigma}{E} + f(\sigma)\ g(t) \tag{58}$$

and this equation is differentiated with respect to σ and is evaluated at the average column stress P/A, the result is

$$\left.\frac{\partial\epsilon}{\partial\sigma}\right]_{\sigma=\frac{P}{A}} = \frac{1}{E} + f'\left(\frac{P}{A}\right) g(t)$$

where f' denotes differentiation with respect to P/A. At $t = t_{cr}$ according to the isochronous-tangent-modulus method, the result is

$$\left.\frac{\partial\epsilon}{\partial\sigma}\right]_{\sigma=\frac{P}{A}} = \frac{1}{E} + f'\left(\frac{P}{A}\right) g\left(t_{cr}\right) = \frac{1}{E_T} \tag{59}$$

where E_T is the tangent modulus on the isochronous curve corresponding to t_{cr} at a stress of P/A. The Euler column formula is given as

$$\frac{P_E}{A} = \frac{\pi^2 E}{\left(\frac{l}{\rho}\right)^2} \tag{60}$$

and the modified Euler formula is given as

$$\frac{P}{A} = \frac{P_T}{A} = \frac{\pi^2 E_T}{\left(\frac{l}{\rho}\right)^2} \tag{61}$$

Equations (59) to (61) can be combined to eliminate E and E_T. The resulting equation is

$$\left(\frac{l}{\pi\rho}\right)^2 \frac{P}{A} \frac{f'\left(\frac{P}{A}\right) g\left(t_{cr}\right)}{1 - \frac{P}{P_E}} = 1 \tag{62}$$

For both the hyperbolic sine creep law and the power creep law, the time parameter is

$$\tau_s \text{ or } \tau_p = \left(\frac{l}{\pi\rho}\right)^2 \frac{P}{A} \frac{f'\left(\frac{P}{A}\right) g(t)}{1 - \frac{P}{P_E}}$$

When $t = t_{cr}$, then $\tau_s = \tau_{s,cr}$ or $\tau_p = \tau_{p,cr}$; thus, the relation

$$\tau_{s,cr} = \tau_{p,cr} = 1 \tag{63}$$

is the column failure criterion for the isochronous-tangent-modulus method.

RESULTS AND DISCUSSION

Calculations

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By using the hyperbolic sine creep law for the solutions of the H-section column and the rectangular column with linear stress distribution, critical lifetime parameters were calculated for a range of lateral-displacement parameters $s(0)$ from less than 0.1 to about 10. These results are plotted in figure 2 which shows a comparison between the rectangular section and its equivalent H-section. For lateral-displacement parameters $s(0)$ less or greater than those shown in figure 2, the lifetime parameters are given by asymptotic formulas derived in the appendix. Calculations of critical time parameters were also made by using the analyses with the power creep law for the equivalent H-section column and the rectangular-section column with linear stress distribution through the thickness. The values of n used were the odd integers from 3 to 9, and the range of displacement parameter $W(0)$ corresponding to the elastic column with load was varied from about 0.001 to 1. Results are given in figure 3.

For rectangular-section columns with nonlinear stress distribution through the thickness, the system of differential equations (eqs. (55) and (56)) was solved numerically on an IBM type 704 electronic data processing machine using the Runge-Kutta method. The power creep law was utilized in this calculation. The required numerical integrations were carried out by using Gaussian quadrature formulas with 10 stations through the thickness. The necessary quadrature coefficients and station locations were obtained from reference 13. Displacements and stresses were calculated as functions of the time parameter. The value of the time parameter at which the displacement rate became essentially infinite was taken as the critical time parameter.

Comparisons Between Various Theories

Based on Power Creep Law

In figures 4 and 5 the critical time parameters for rectangular columns and nonlinear stress distribution are given as the circled and squared points, and these nonlinear-stress results are compared with those for linear stress and for the equivalent H-section column. All these calculations are based on the power creep law. Also given as the uniform dash curve is an approximate nonlinear stress solution in which the initial stress distribution is not quite linear. The uniform dash curves were obtained from a single calculation of W as a function of τ_p for each value of P/P_E starting with linear initial stress at a very small value of $W(0)$ $(W(0) = 0.001)$. As an approximation, the resulting stress conditions existing when W reached a given value were assumed to be the initial conditions for the particular column starting out with a value of $W(0)$ equal to that given value of W. The approximation is very good except when both $W(0)$ and n are large.

The ratio of the critical time parameter for the nonlinear-stress solution to that for the linear-stress solution is given in figure 6 based on the power creep law. The individual data points were calculated by the more exact method discussed previously in which the initial stresses are linear. The curves were arbitrarily faired through the points. Except for large values of n and $W(0)$ the effect of the nonlinear stress distribution on column creep life is negligible. Thus, for many cases an analysis based on the assumption of a linear stress distribution should be sufficient.

Typical Stress Distributions

In figure 7 are shown some typical stress distributions through the thickness of the rectangular-section column based on the power creep law. The symbol σ_B represents the difference between the stress in the column at a particular value of z/h and the average stress on the column. In order to emphasize the deviations from a linear distribution, the curves are normalized with respect to the maximum value of σ_B calculated by elementary beam theory for the value of W indicated. These particular stress distributions were obtained for $W(0) = 0.001$, and the dashed-line curves indicate the initial stress distribution at $W(0) = 0.001$, which is linear.

Column Lifetimes and Material Creep Data

Comparisons were made to determine the correlation to be expected between column lifetimes calculated with different creep laws matched to the same material creep curves. Constants for both the hyperbolic sine law and the power law were obtained from compression creep data in reference 1 on 7075-T6 aluminum alloy at 600° F. These constants are listed in table 1. Both creep laws agree closely with the data which are in the stress range from 4.5 to 5.5 ksi. It is characteristic of these laws for the hyperbolic sine law to give the greater creep strain outside the stress range of the data (either above or below).

Column lifetimes were calculated by using both creep laws for rectangular-section columns with linear stress distribution through the thickness, and the results are given in table 2. The material creep data were in the stress range from 4.5 to 5.5 ksi, as mentioned previously; and the two calculations agree very closely for average column stresses between 4 and 5 ksi.

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For theoretical column creep analyses to be accurate, the creep law should be accurate throughout the range of the stress which the column experiences. Data are not ordinarily available from which a creep law can be formulated that is accurate throughout such a wide stress range. The calculated results in table 2 suggest that when only limited material creep data are available, the best results for calculated lifetime may be obtained when the range of material creep data covers the average column stress or is slightly above it.

Comparison Between Theory and Experiment

Column stresses for various creep lifetimes calculated from theory are compared with experiment in figures 8 to 11. Four sets of material and column data given in references 1 to 4 were considered in this comparison. The material creep data and column data have been screened so that only those columns whose average stress fell in a certain range were included. The stress range for the column data was taken to be that between the highest stress in the corresponding material creep data and a value about two-thirds of the lowest stress in the material data. All the available column tests in this range were included provided there was more than one test for a given value of l/ρ. Material constants and the range of the data from which they were obtained are given in table 1. The theories used for the comparison include the rectangular-section column both with linear stress distribution and with nonlinear stress distribution through the thickness. Also included is the isochronous-tangent-modulus method. Refinements in the theory have relatively little effect on the theoretical results. Furthermore, the refinements do not always lead to an improvement in the comparisons. In many cases, the results obtained from the isochronous-tangent-modulus method appear to be about as good as the results obtained from the theoretical methods.

CONCLUDING REMARKS

A theoretical analysis has been made of idealized H-section and rectangular-section columns by the application of a creep variational theorem. For the rectangular-section columns both linear and nonlinear stress distributions through the thickness are considered. The solution for the case of nonlinear stress distribution utilizes a power creep law; the other solutions are given for both the power law and the hyperbolic sine creep law. Charts for critical lifetime parameters are given.

Comparisons were made between the theoretical solutions for the rectangular column and previously published experimental data. The isochronous-tangent-modulus method was also included in the comparisons. The results indicate that the differences between the linear-stress and
nonlinear-stress solutions are rather small, a linear-stress solution L
being sufficient in most cases. Refinement in the stress distribution 3
does not always improve the comparisons between theory and test and, 3
thus, does not appear worthwhile at present. Finally, in many cases, 8
the results obtained from the isochronous-tangent-modulus method were about as good as the results obtained from the more involved theoretical methods.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Field, Va., May 21, 1959.

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APPENDIX

ASYMTOTIC FORMULAS FOR EXTREME VALUES OF INITIAL LATERAL-DISPLACEMENT PARAMETER

When the argument is sufficiently small, the modified Bessel function $I_1(x)$ can be approximated by $I_1(x) \approx \frac{x}{2}$ (ref. 14). The integrals in equations (20) and (36) can each be written as the sum of two integrals - one from $s(0)$ or $s_1(0)$ to s^* and another from s^* to ∞. If s^* is small enough for the approximation for $I_1(x)$ to hold with fair accuracy, the following simplified expression for critical time results:

$$\tau_{s,cr} = \log_e \frac{s^*}{s} + \tau_{s,cr}(s^*) \qquad \text{(A1)}$$

where s is replaced by s_1 in the case of the idealized H-section column. Numerical calculations indicate that sufficient accuracy results when $s^* = 1$. Approximate expressions for the critical lifetime parameter are given for the idealized H-section column by

$$\tau_{s,cr} = \log_e \frac{1}{s_1} + 1.37 \qquad (s_1 < 1) \qquad \text{(A2)}$$

and for the rectangular-section column by

$$\tau_{s,cr} = \log_e \frac{1}{s} + 1.04 \qquad (s < 1) \qquad \text{(A3)}$$

When the argument is large, the Bessel function approaches (see ref. 14)

$$I_1(x) \approx \frac{e^x}{\sqrt{2\pi x}} \qquad \text{(A4)}$$

The use of this approximation to calculate the lifetime parameter for H-section columns having large values of initial crookedness gives

$$\tau_{s,cr} \approx \sqrt{\frac{\pi}{2}} \int_{s_1}^{\infty} x^{1/2} e^{-x} dx \qquad \text{(A5)}$$

Making the substitution $z = x^{1/2}$ and integrating by parts gives

$$\tau_{s,cr} \approx \sqrt{\frac{\pi}{2}}\left(s_1{}^{1/2}e^{-s_1} + \int_{s_1{}^{1/2}}^{\infty} e^{-z^2}dz\right) \tag{A6}$$

The integral remaining in equation (A6) is

$$\int_{s_1{}^{1/2}}^{\infty} e^{-z^2}dz = \int_0^{\infty} e^{-z^2}dz - \int_0^{s_1{}^{1/2}} e^{-z^2}dz = \frac{\sqrt{\pi}}{2}\left[1 - \Phi\left(s_1{}^{1/2}\right)\right] \tag{A7}$$

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where $1 - \Phi\left(s_1{}^{1/2}\right)$ is the complementary error function. The asymptotic expansion for $1 - \Phi\left(s_1{}^{1/2}\right)$ is

$$1 - \Phi\left(s_1{}^{1/2}\right) \approx \frac{2}{\sqrt{\pi}} \frac{e^{-s_1}}{2s_1{}^{1/2}}\left(1 - \frac{1}{2s_1} + \ldots\right) \tag{A8}$$

Thus, the critical lifetime parameter becomes

$$\tau_{s,cr} \approx \sqrt{\frac{\pi}{2}}\, e^{-s_1}\left(s_1{}^{1/2} + \frac{1}{2}\, s_1{}^{-1/2}\right) \tag{A9}$$

This approximation is small by less than 4 percent when $s_1 = 10$ and is more accurate for larger values of s_1.

For the rectangular-section column having large values of initial crookedness and linear stress distribution through the thickness, the approximate form for the critical lifetime parameter becomes

$$\tau_{s,cr} \approx \frac{1}{6} \int_s^{\infty} \frac{\gamma^2 d\gamma}{\displaystyle\int_0^{\gamma} \frac{\beta^{1/2}e^{\beta}}{\sqrt{2\pi}}\, d\beta} \tag{A10}$$

The integral in the denominator can be approximated for a very large value of γ by

$$\int_0^{\gamma} \frac{\beta^{1/2}e^{\beta}}{\sqrt{2\pi}}\, d\beta \approx \frac{e^{\gamma}\gamma^{1/2}}{\sqrt{2\pi}} \tag{A11}$$

A numerical integration with $\gamma = 9.95$ shows that this approximation is about 12 percent high and improves for larger values of γ. When equation (A11) is substituted into equation (A10) and an integration by parts is carried out, the result is

$$\tau_{s,cr} \approx \frac{\sqrt{2\pi}}{6}\left(s^{3/2}e^{-s} + \frac{3}{2}\int_{s}^{\infty} \gamma^{1/2}e^{-\gamma}d\gamma\right) \tag{A12}$$

The integral in equation (A12) can be handled in the same manner as that in equation (A5). The result can be approximated as

$$\tau_{s,cr} \approx \frac{\sqrt{2\pi s}}{4}\, e^{-s}\left(\frac{2}{3}\, s + 1\right) \tag{A13}$$

This expression is low by about 10 percent for $s = 10$.

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TABLE 1

COMPRESSION CREEP CONSTANTS OF TWO ALUMINUM ALLOYS FOR THE CREEP LAWS USED

$$\left[\epsilon_p = \frac{\sigma}{E} + \left(\frac{\sigma}{\sigma^*}\right)^n t^k;\ \epsilon_s = \frac{\sigma}{E} + C \sinh B\sigma t^k\right]$$

Ref.	Material	T, °F	E, ksi	C, $\frac{1}{(hr)^k}$	B, $(ksi)^{-1}$	k	n	σ*, $(ksi)(hr)^k$	Stress range, ksi	Time range, hr
1	7075-T6	600	5.2×10^3	6.0×10^{-7}	1.90	0.60	9.54	8.90	4.5 to 5.5	0 to 16
3	2024-T4, stabilized[a]	450	9.1	4.7×10^{-6}	.85	.26	6.3	21.3	7.0 to 8.0	0 to 160
4	2024-T4, as received	350	10.1	2.28×10^{-6}	.183	.50	3.25	412	12 to 28	0 to 160
4	2024-T4, as received	450	9.0	5.54×10^{-5}	.122	.60	2.07	980	12 to 20	0 to 160

[a]Material stabilized at 600° F for 100 hours.

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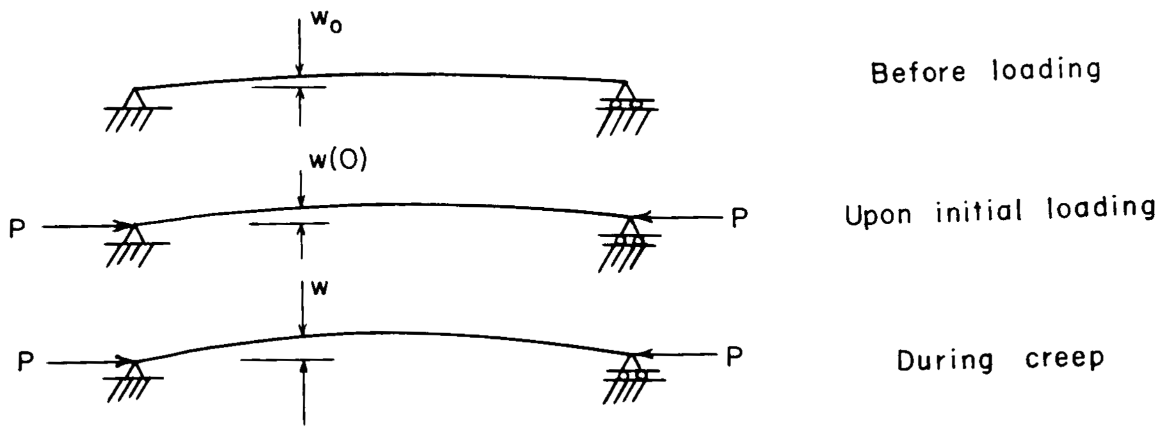
TABLE 2

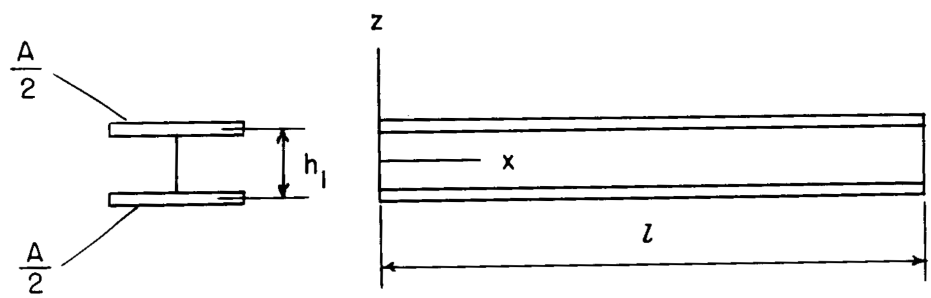
COMPARISON BETWEEN CALCULATED LIFETIMES OF RECTANGULAR-SECTION COLUMNS FOR TWO DIFFERENT CREEP LAWS

[Linear stress distribution through thickness; material, 7075-T6 aluminum alloy in compression at 600° F; stress range for material creep data, 4.5 to 5.5 ksi]

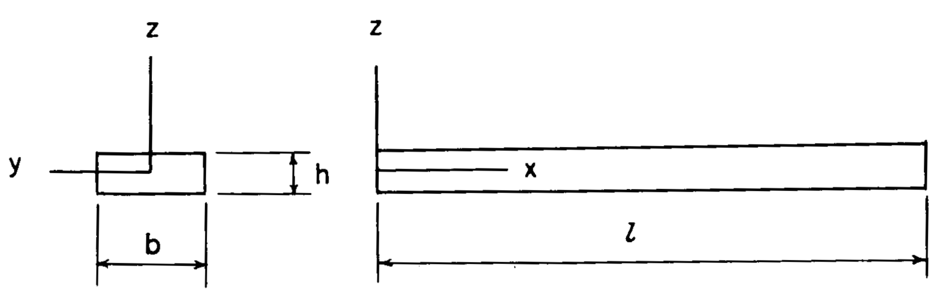
W_o	$\frac{l}{\rho}$	$\frac{P}{A}$, ksi	Power law	Hyperbolic sine law
			$\tau_{p,cr}$, hr	$\tau_{s,cr}$, hr
0.01	30	5.5	0.056	0.034
.01	30	5	.255	.222
.01	70	4	.188	.195
.01	100	3	1.74	.992
.01	120	2.5	5.35	1.70



(a) Symbols for lateral displacement of middle surface of column.



(b) Idealized H-section column.



(c) Rectangular-section column.

Figure 1.- Symbols for lateral displacement and coordinate systems.

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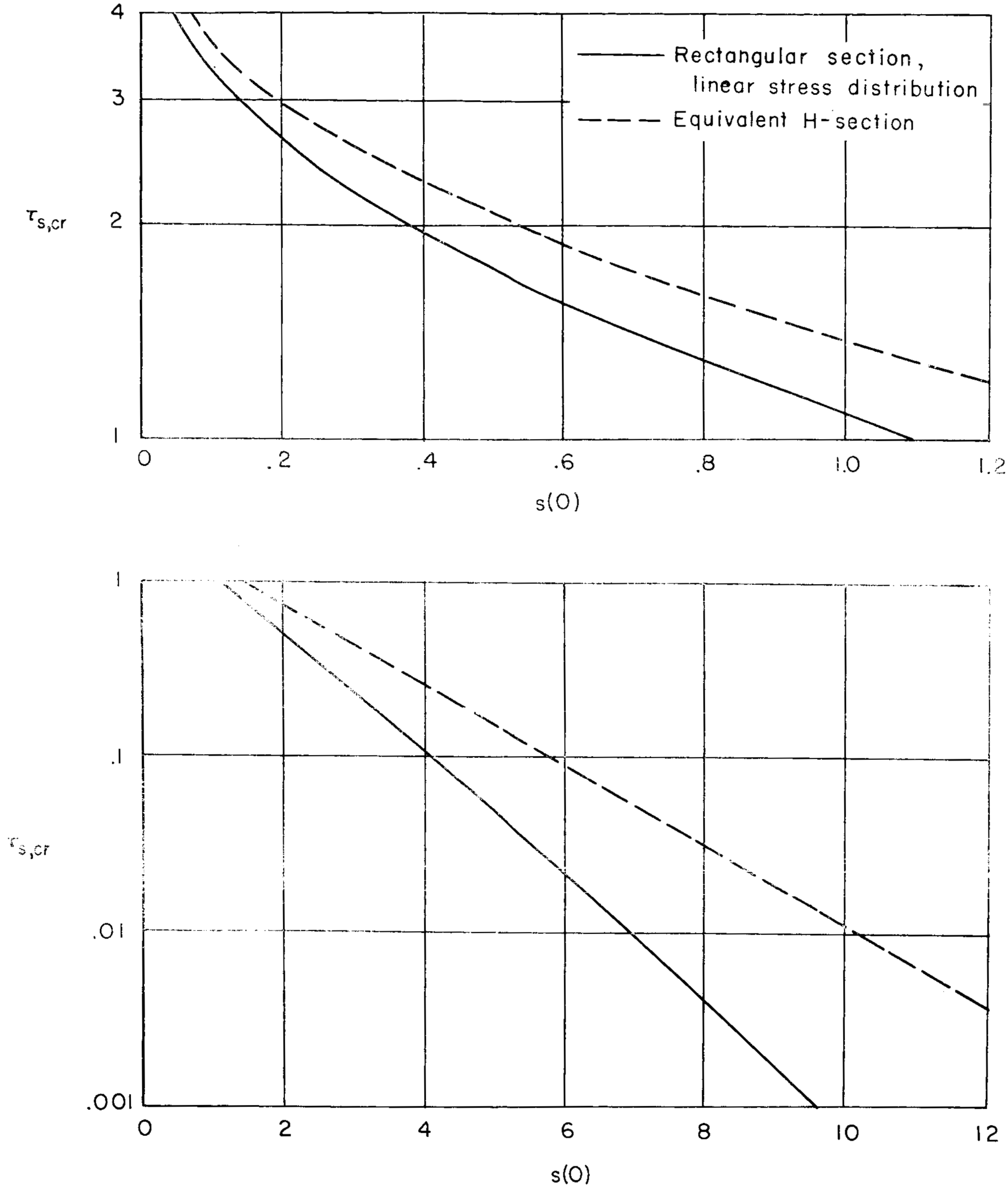

Figure 2.- Column lifetime parameter from variational theorem using hyperbolic sine creep law.

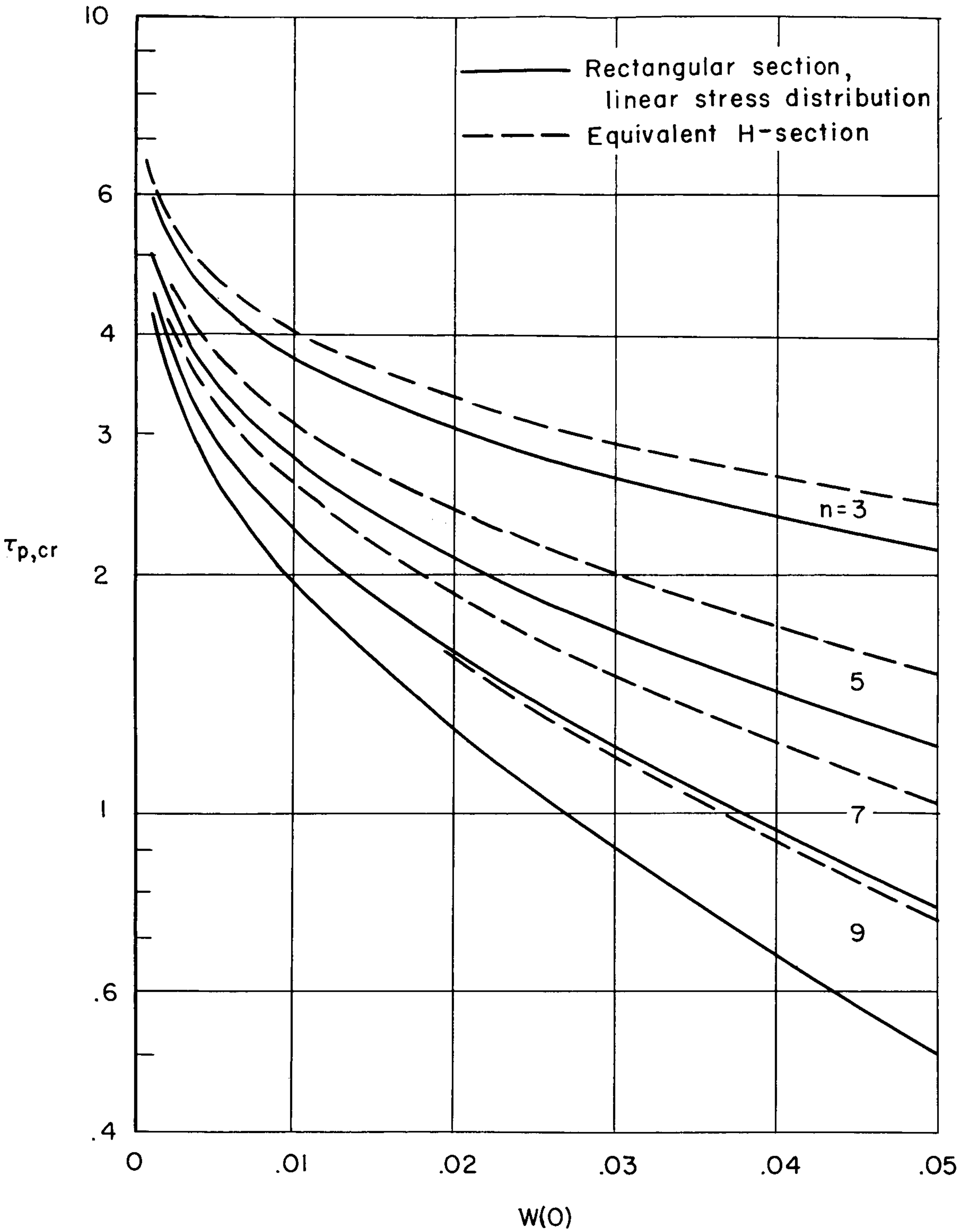

Figure 3.- Column lifetime parameter from variational theorem using power creep law.

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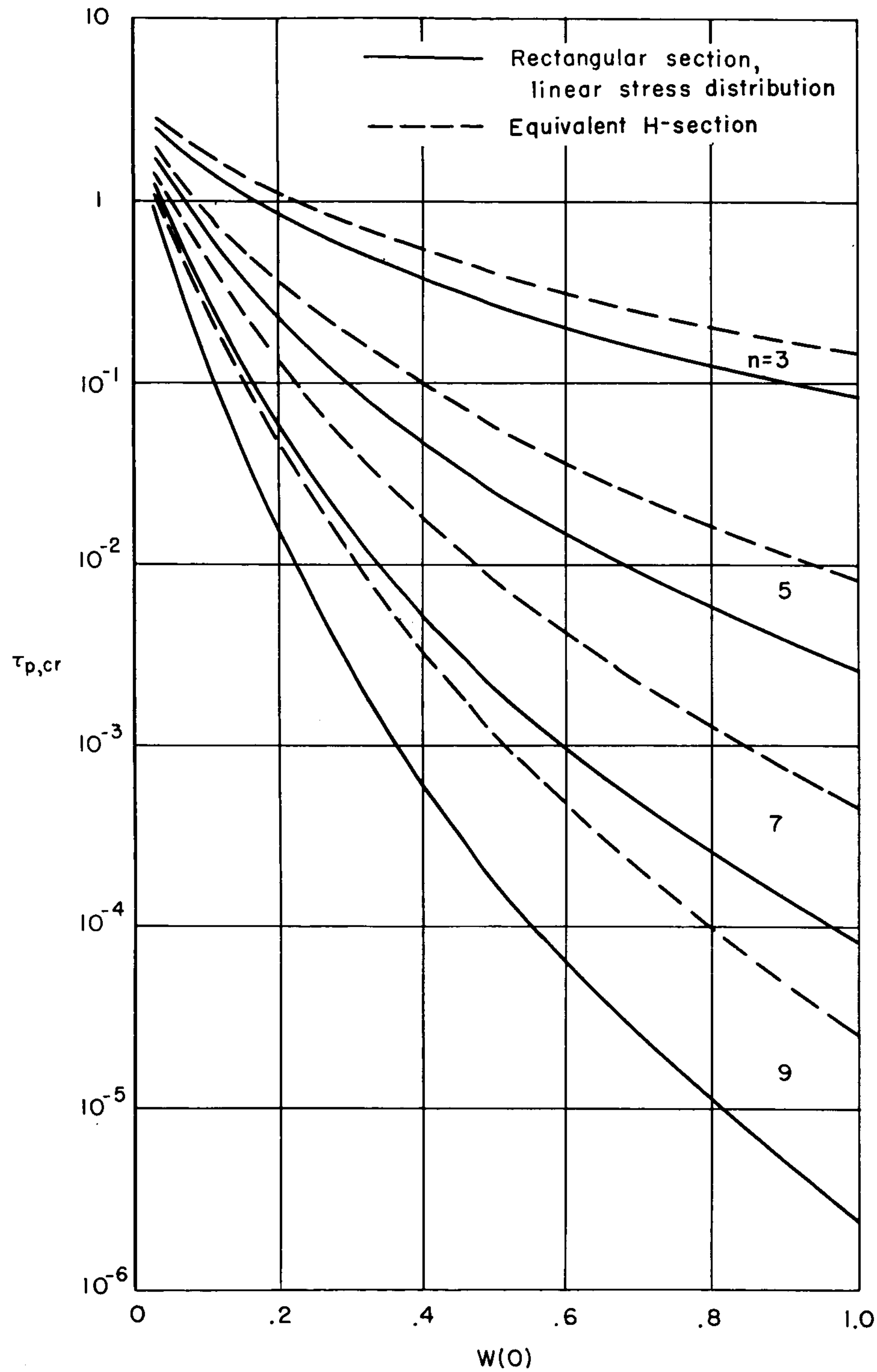

Figure 3.- Concluded.

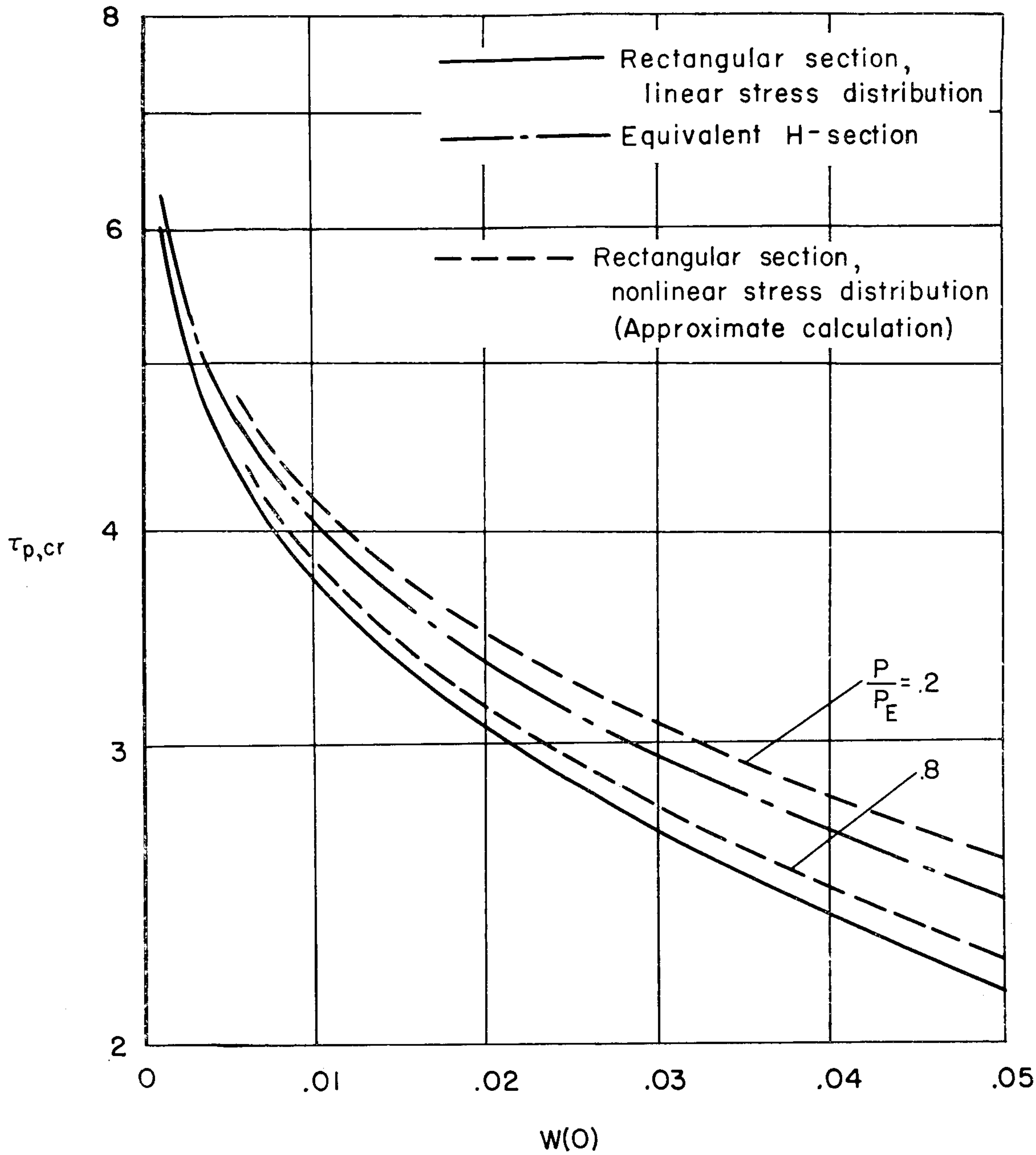

Figure 4.- Lifetime parameters calculated by three different theories for rectangular-section column. Power creep law; n = 3.

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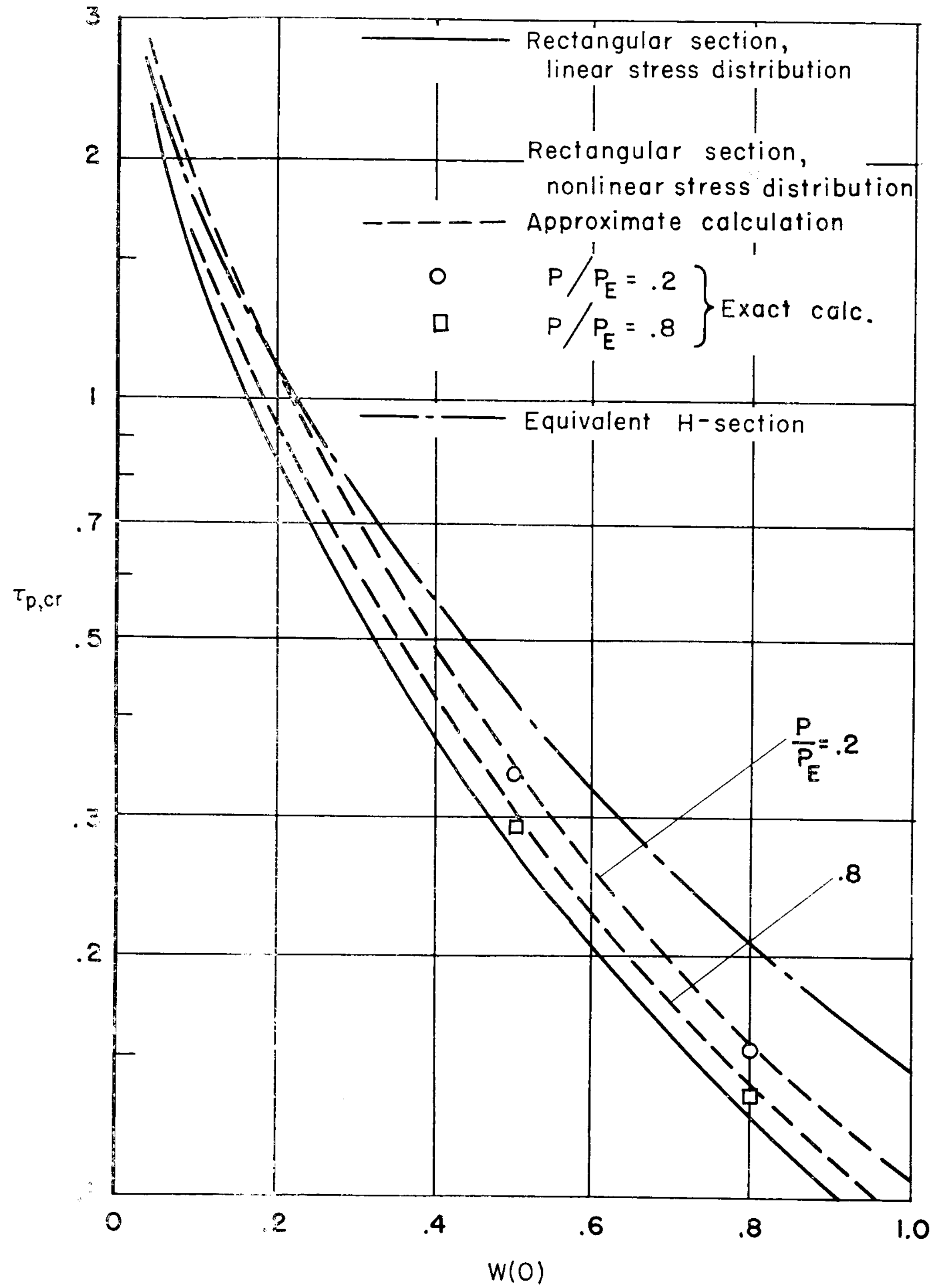

Figure 4.- Concluded.

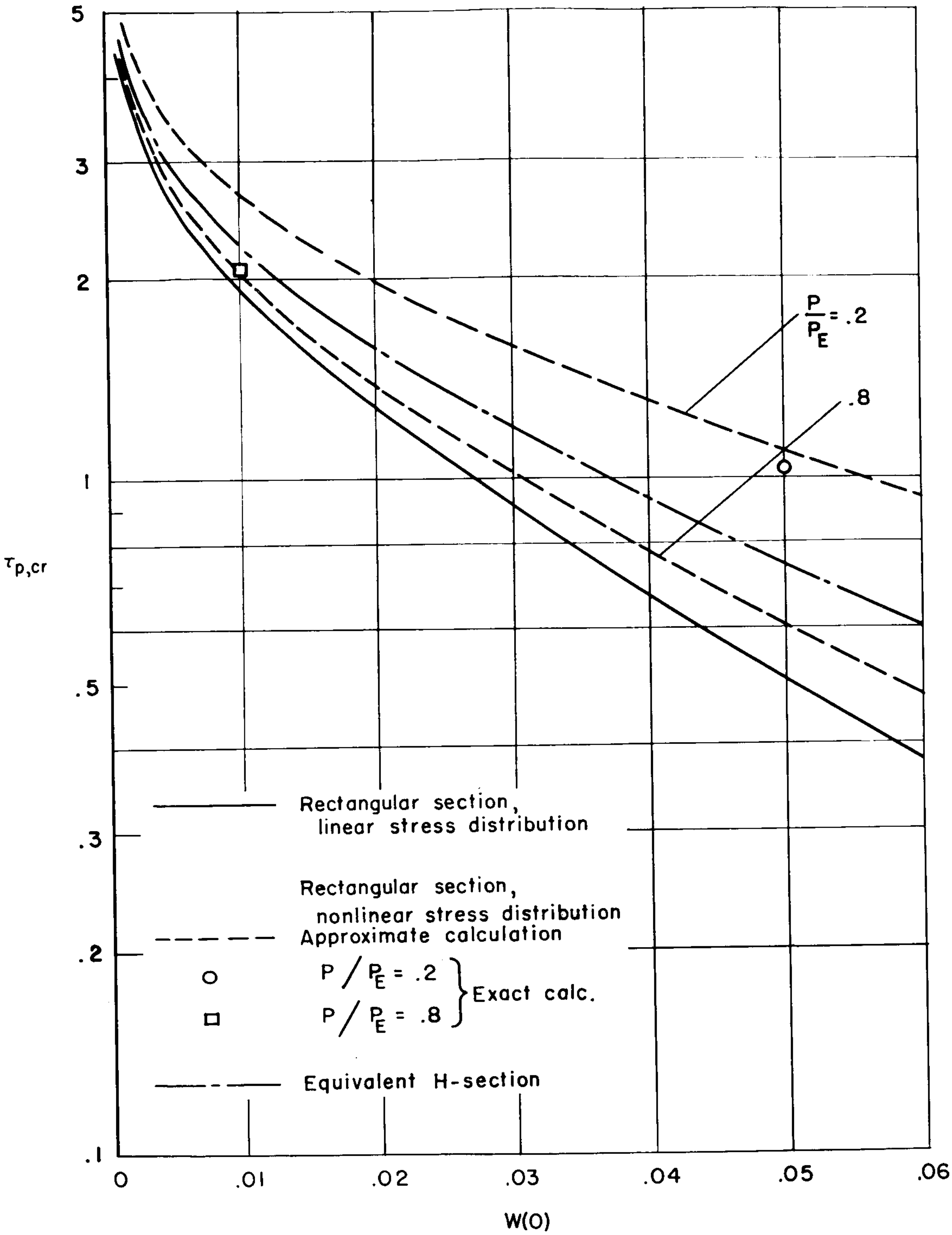

Figure 5.- Lifetime parameters calculated by three different theories for rectangular-section column. Power creep law; n = 9.

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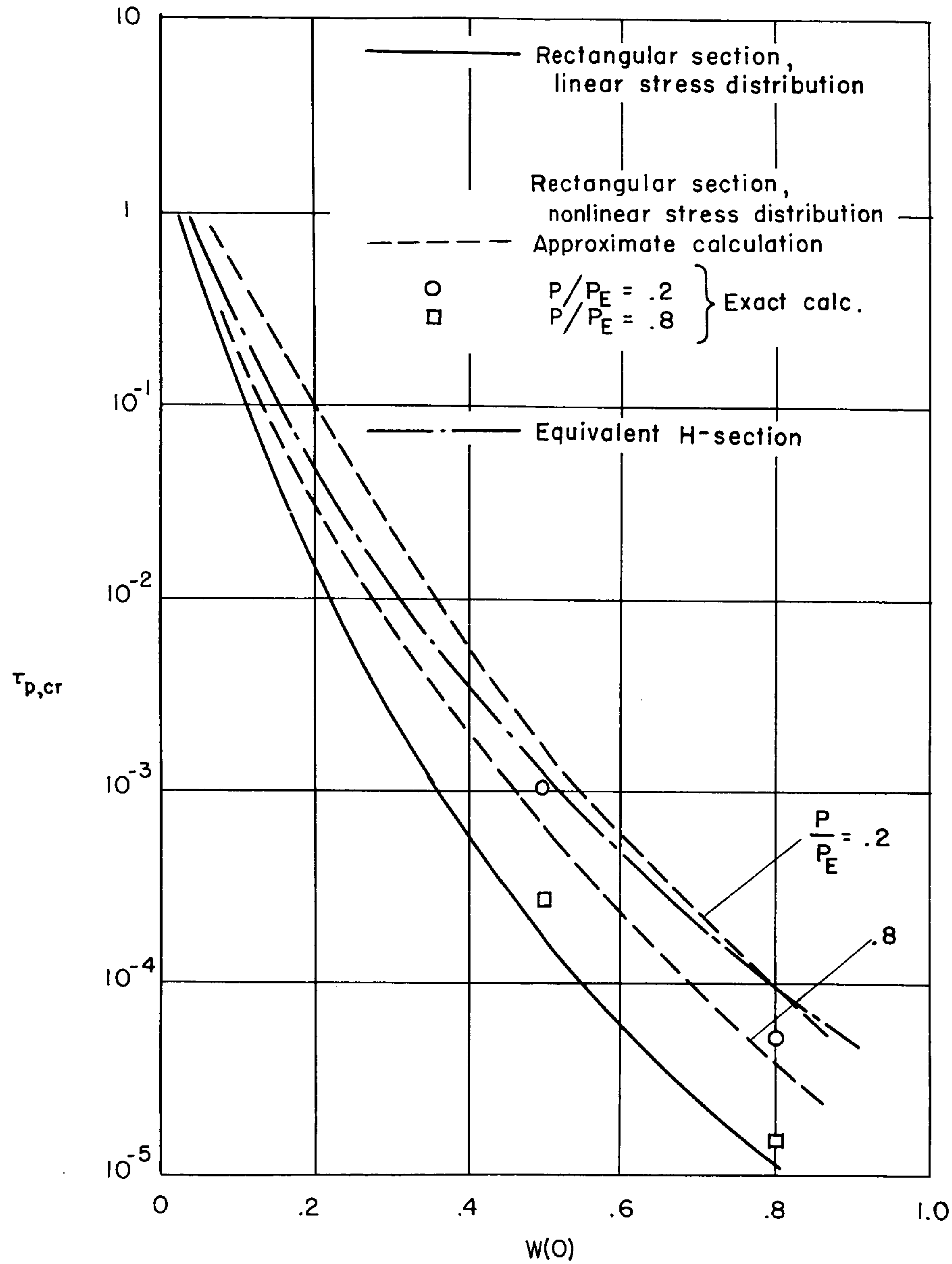

Figure 5.- Concluded.

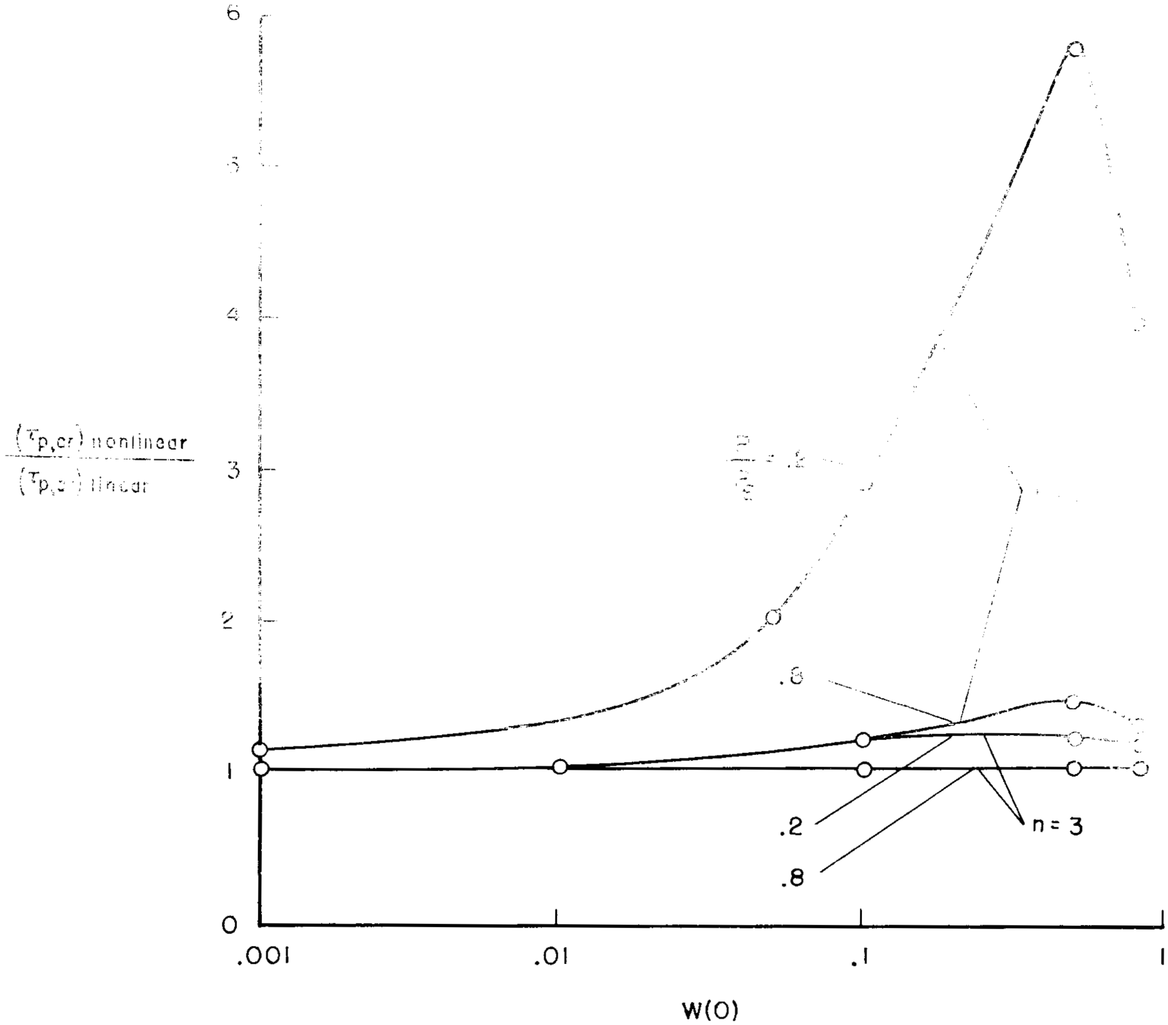

Figure 6.- Ratio of lifetime parameter for nonlinear-stress theory (initial stresses linear) to lifetime parameter for linear-stress theory for rectangular-section columns. Power creep law.

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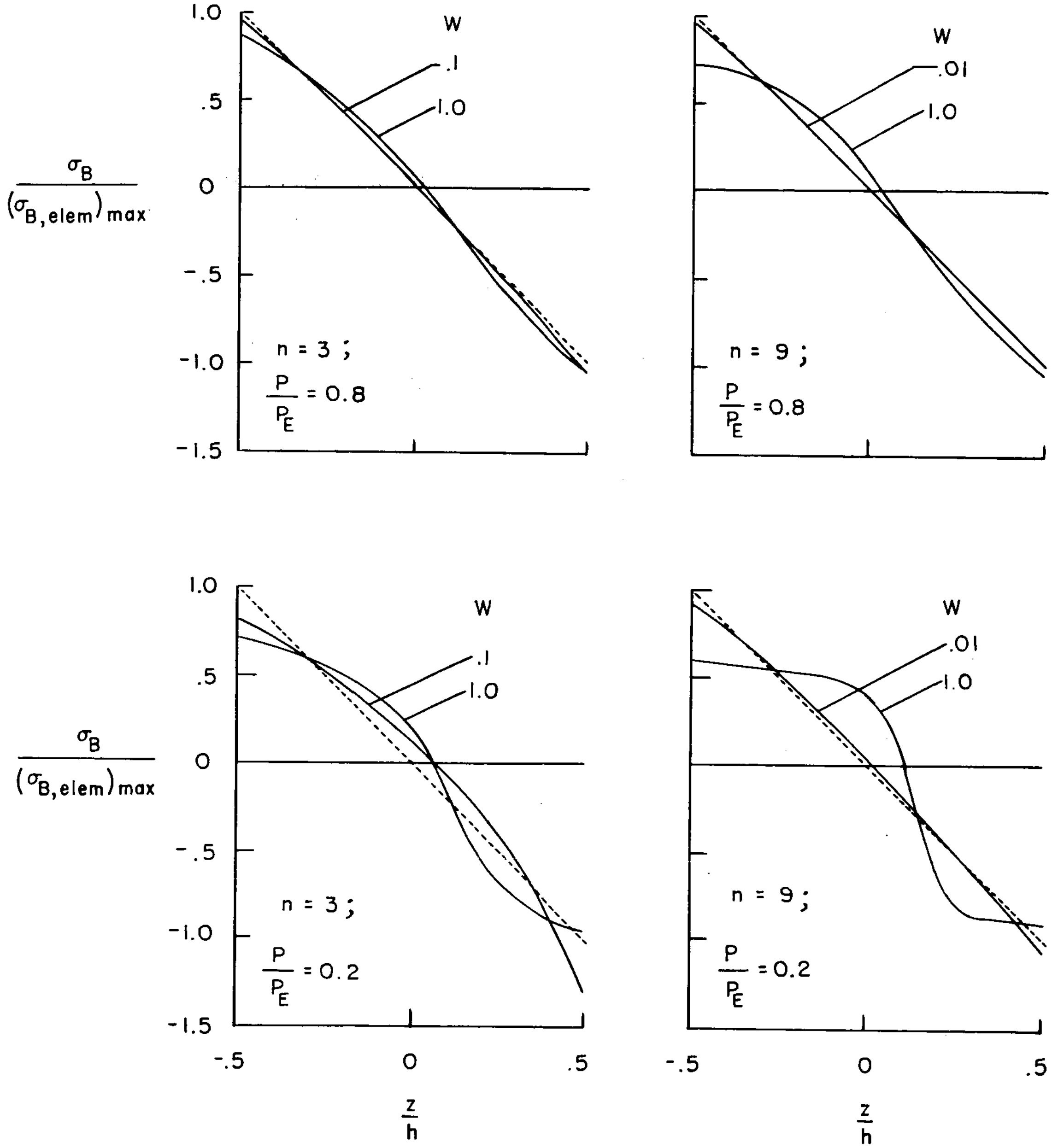

Figure 7.- Typical stress distributions for rectangular-section columns. Power creep law; W(0) = 0.001.

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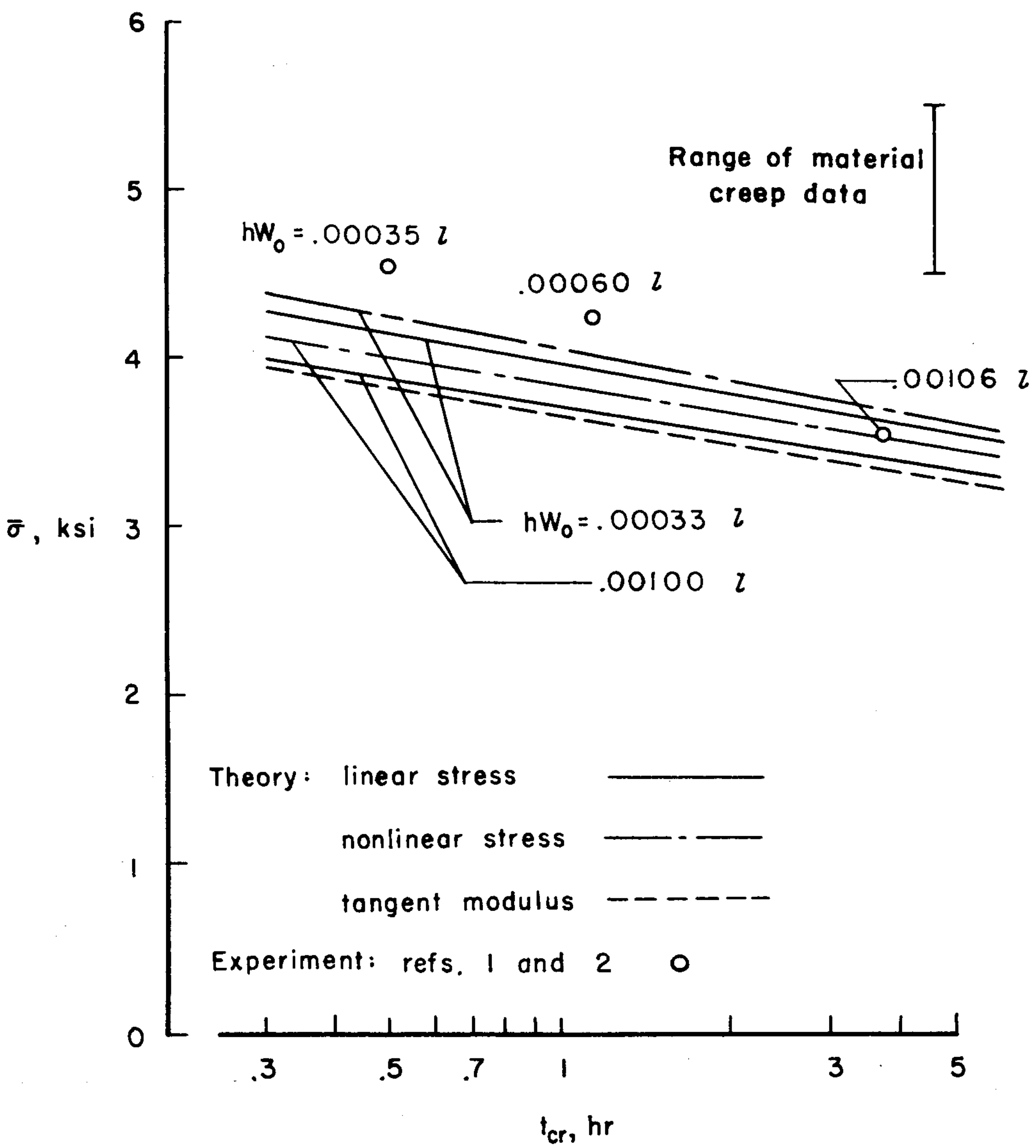

(a) $\frac{l}{\rho} = 58.7$.

Figure 8.- Comparison between calculated and experimental lifetimes for rectangular-section columns made of 7075-T6 aluminum alloy. T = 600° F.

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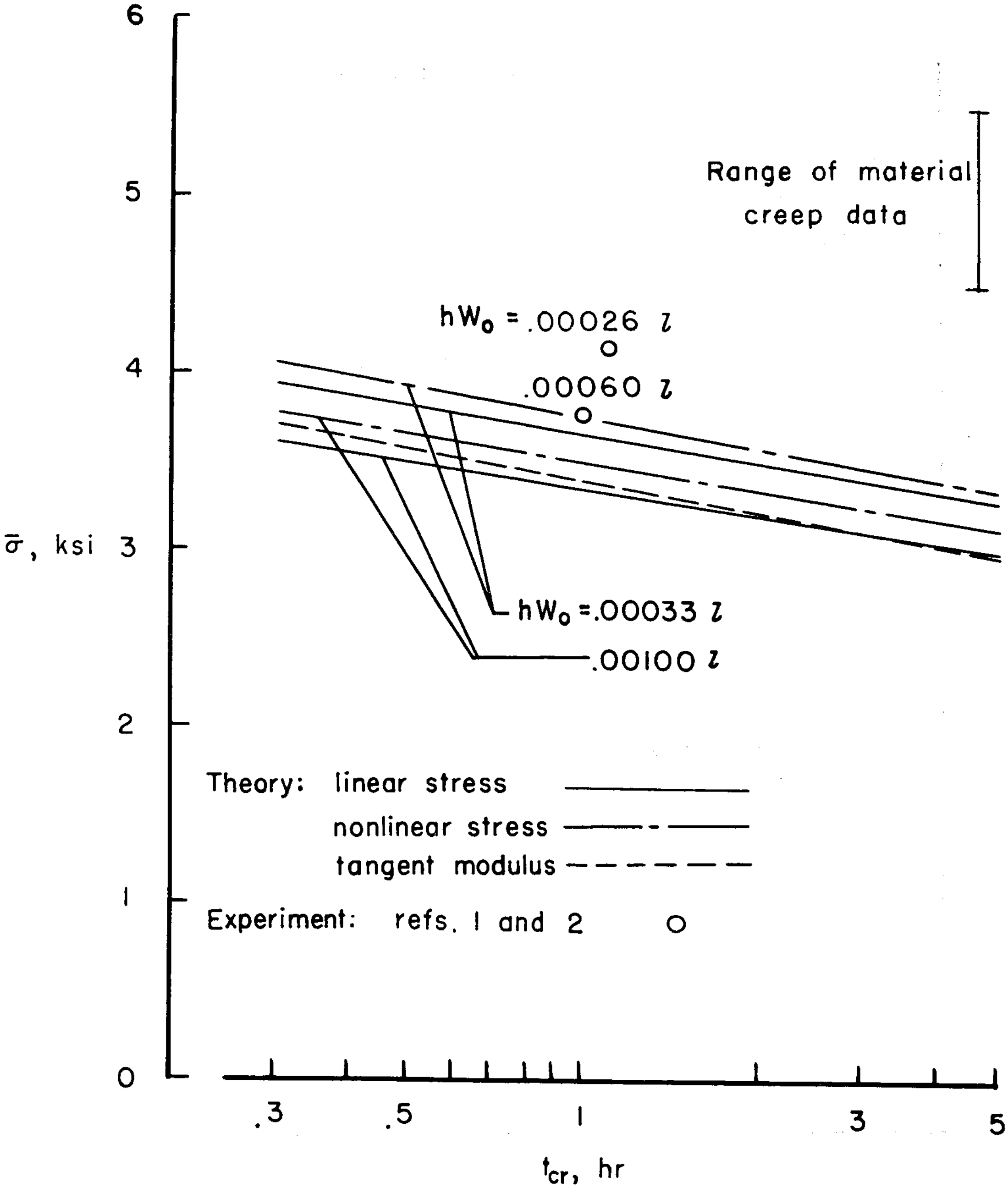

(b) $\frac{l}{\rho}$ = 72.6.

Figure 8.- Concluded.

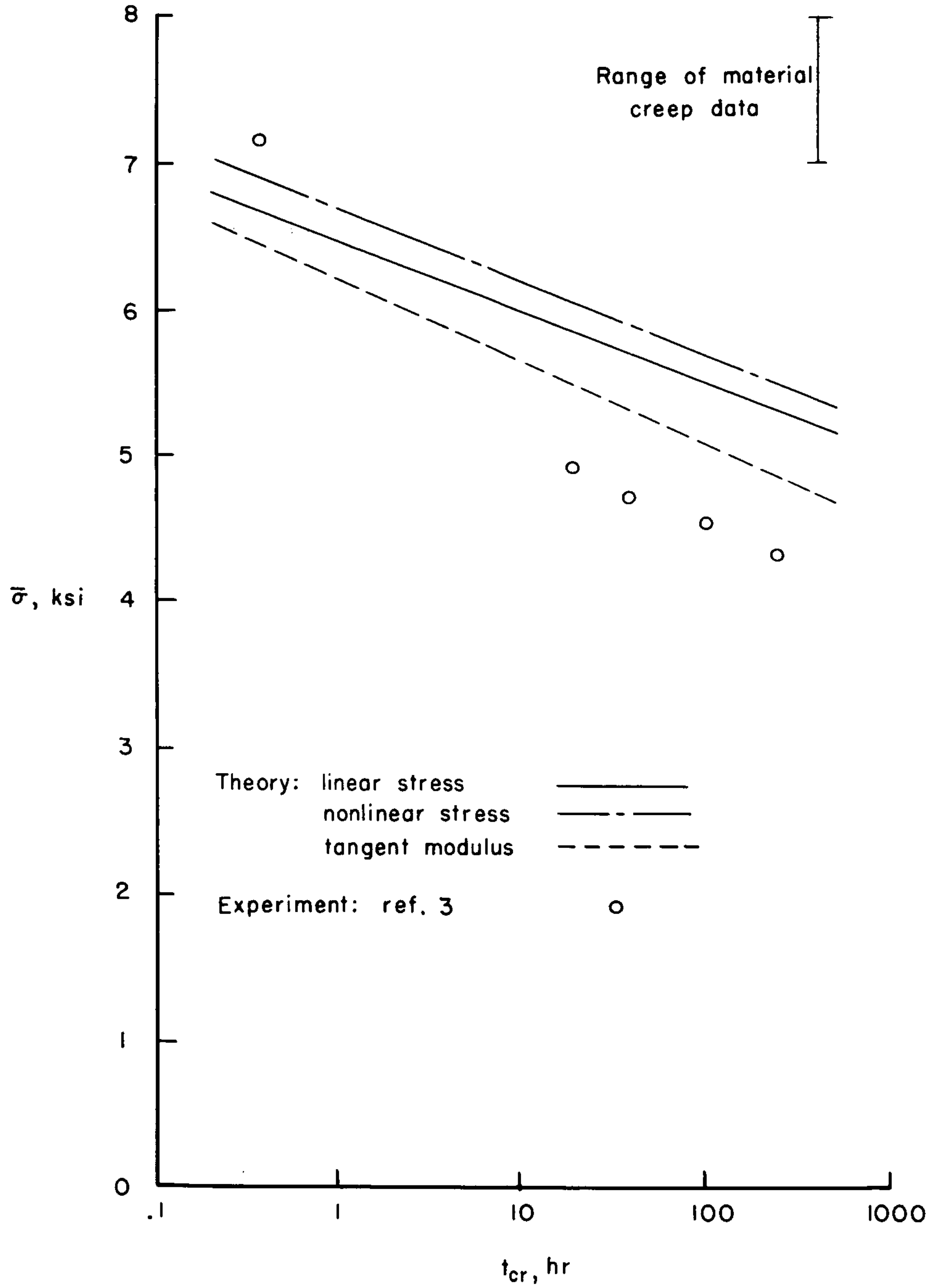

Figure 9.- Comparison between calculated and experimental lifetimes for rectangular-section columns made of stabilized 2024-T4 aluminum alloy. $T = 450^\circ$ F; $\frac{l}{\rho} = 56.5$; $hW_o = 0.00111l$.

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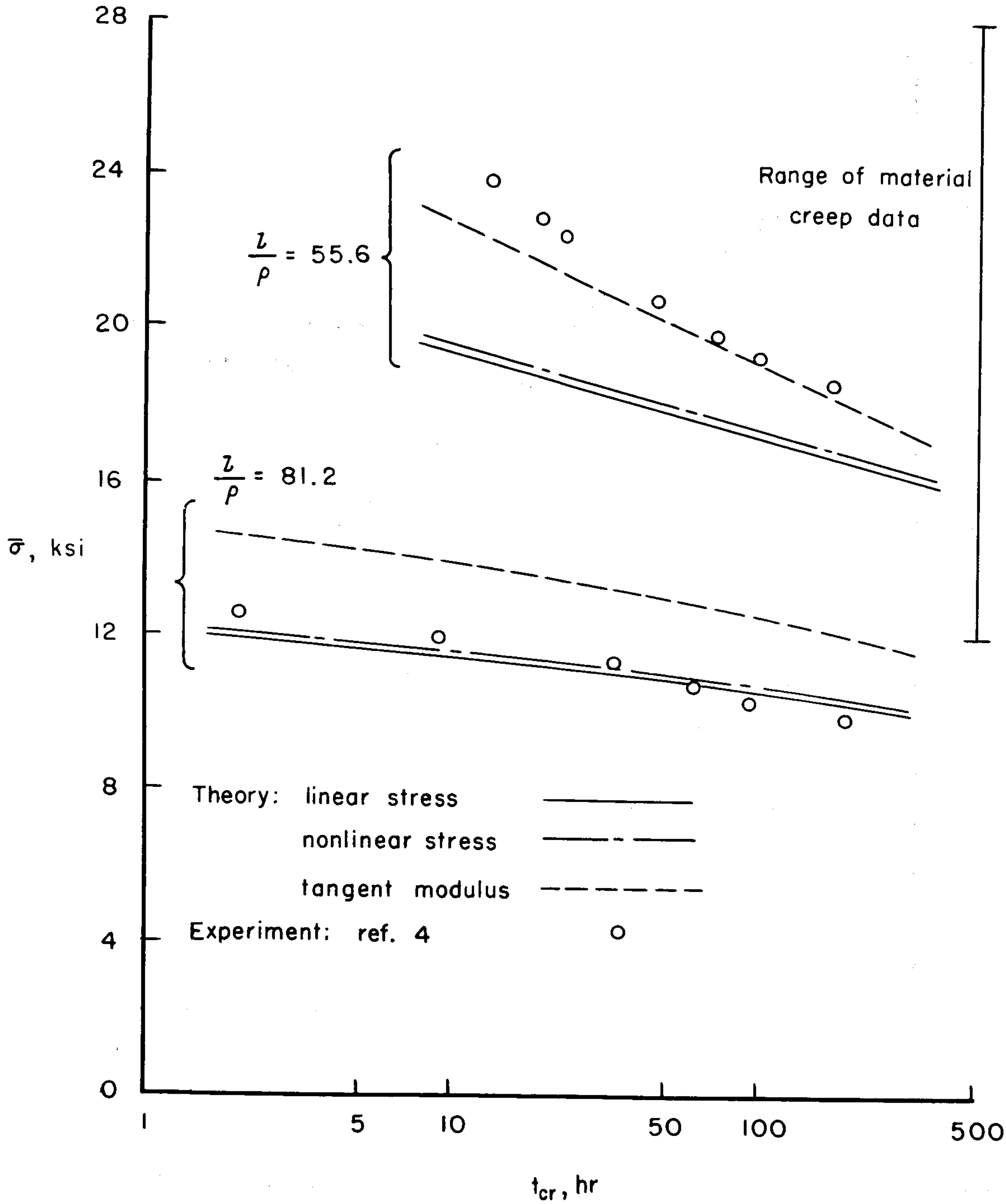

Figure 10.- Comparison between calculated and experimental lifetimes for rectangular-section columns of 2024-T4 aluminum alloy, as received. T = 350° F; $hW_o = 0.00286l$.

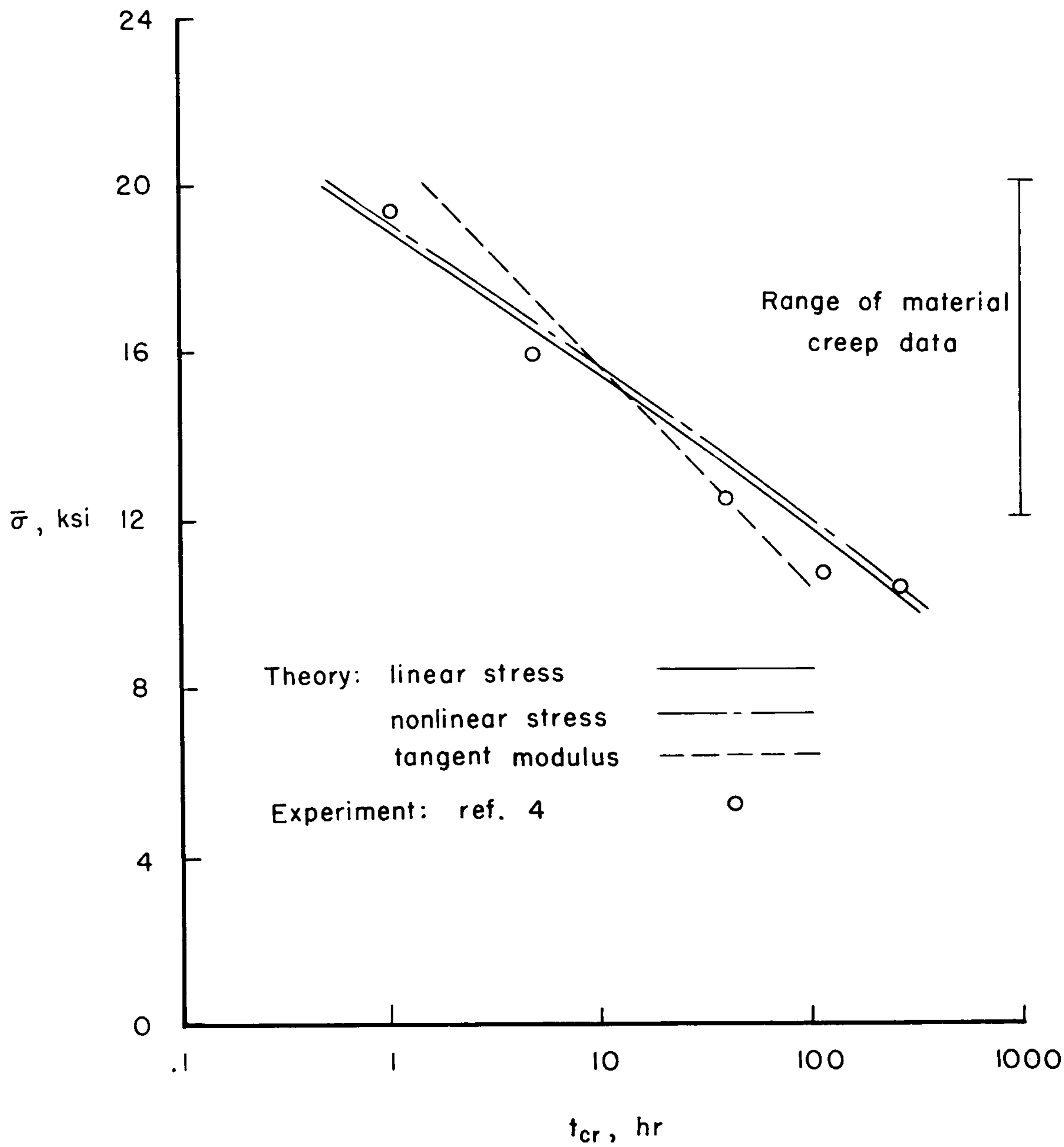

Figure 11.- Comparison between calculated and experimental lifetimes for rectangular-section columns of 2024-T4 aluminum alloy, as received. $T = 450^{\circ}$ F; $\frac{l}{\rho} = 55.6$; $hW_o = 0.00286l$.